1. **PROBLEM:** Consider the function

$$g(x) = \int_1^x f(t)\ dt$$

If $f(t)$ has a **positive derivative** for all values of t and $f(2) = 0$, decide which of the following statements are **always true**, which are **always false** and which you have **not enough information** to make a claim. There's no need to justify your claims. Simply write on your blue book the statement letter along with your claim: TRUE, FALSE or NOT ENOUGH INFO.

   (a) $g(x)$ is a differentiable function of x.
   (b) $g(x)$ is a continuous function of x.
   (c) The graph of $g(x)$ has a horizontal tangent at $x = 2$.
   (d) $g(x)$ has a local maximum at $x = 2$.
   (e) The graph of $g(x)$ has an inflection point at $x = 2$.

2. **SOLUTION:**

   (a) Since $f(t)$ has a positive derivative for all values of t, its derivative exists for all values of t. Therefore, $f(t)$ is continuous everywhere. Thus, we can use the Fundamental Theorem of Calculus to compute the derivative of $g(x)$ with respect to x, getting that $g'(x) = f(x)$, which exists for all values of x. Therefore, $g(x)$ is a differentiable function of $x \Rightarrow$ (a) is **always true**.
   (b) From part(a) above, we know that $g(x)$ is differentiable everywhere. Therefore, it must be continuous everywhere $\Rightarrow$ (b) is **always true**.
   (c) From part(a) above, we know that $g'(x) = f(x)$. Thus, $g'(2) = f(2) = 0$, which implies that a tangent line to $g(x)$ at $x = 2$ has a zero slope. Therefore, such tangent line is horizontal $\Rightarrow$ (c) is **always true**.
   (d) Since $g'(x) = f(x)$, we have that $g''(x) = f'(x)$, which from the problem statement is always positive. Since the second derivative of $g(x)$ is always positive, the graph of $g(x)$ is concave up everywhere, which implies that there cannot be any local maximum anywhere $\Rightarrow$ (d) is **always false**.
   (e) A point of inflection is a point where the function has a tangent line and changes concavity. From part(d), we know that $g(x)$ is concave up everywhere. Therefore, it does not change concavity anywhere, so, it has no inflection points $\Rightarrow$ (e) is **always false**.