APPM 1350 FINAL EXAM Summer 2004

ON YOUR FIRST PAGE PLEASE INCLUDE: (1) your name, (2) last 4 digits of your student ID number, and (3) a grading table.

There are seven problems on this exam. You must work questions 1 and 2. In addition, you must work 3 of the last 5 questions. Please cross out the problems that you did not do on your grading table. Only 5 problems will be graded. Note that this exam is worth 150 points. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Unless otherwise specified, please keep your answers in fractional or radical form if applicable (no decimals). Text books, class notes, crib sheets and calculators are NOT permitted. Please start each problem on a new page. GOOD LUCK and DON'T FORGET CONSTANTS & UNITS!!!

1. (30 points) Find **5** of the following **6** integrals.

   (a) $\int \tan(x)\sec^2(x)\,dx$

   (b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

   (c) $\int_{-1}^{1} r\sqrt{1-r^2}\,dr$

   (d) $\int \frac{z^3}{\sqrt{z^4+9}}\,dz$

   (e) $\int \cot(x)\,dx$

   (f) $\int_e^b \frac{1}{x\ln x}\,dx$

2. (30 points) Fact: In the preservation of food, cane sugar is broken down at a rate proportional to the concentration $y(t)$ of unaltered sugar.

   (a) Express the above relation as an equation.

   (b) Letting y_0 represent the initial concentration, solve the equation in part a to find an expression for $y(t)$. Also, explain what each of the terms represent *in the context of this problem*. NOTE 1: Showing the equation without any work will earn partial credit. NOTE 2: Be specific when identifying your terms. For example, you know y_0 is the initial concentration but you should state what it is the initial concentration of.

   (c) If there is half of the original amount of sugar left after 1 week and one quarter of the original amount of sugar left after 2 weeks, how long did it take for 10% of the sugar to break down?
   [LEAVE YOUR ANSWER IN TERMS OF LOGARITHMS]

COOL, THERE'S MORE!! TURN THE PAGE OVER

WORK 3 OF THE FOLLOWING 5 PROBLEMS

3. (30 points) A line shines from the top of a pole 50 feet high. A ball is dropped from the same height from a point 30 feet away from the light. How fast is the shadow of the ball moving along the ground $\frac{1}{2}$ second later? (Assume the ball falls a distance $s = 16t^2$ feet in t seconds.)

4. (30 points) For $a - b$ calculate $\frac{dy}{dx}$. For parts $c - e$ evaluate the following limits if they exist, if they don't exist or are infinite please indicate ∞, $-\infty$, or DNE.

   (a) $y = \sin^{-1}\left(e^{x^2}\right)$

   (b) $y = \ln[\ln(\sin(x))] + \sec\left(\dfrac{\pi}{6}\right)$

   (c) $\displaystyle\lim_{x\to 1} \frac{x-1}{\sin x}$

   (d) $\displaystyle\lim_{x\to 1} \frac{1}{x-1}\int_1^{x^3} \ln(t)\,dt$

   (e) $\displaystyle\lim_{t\to 0} \frac{\sin^3(2t)}{t}$

5. (30 points) The Boulder Rec Center would like to build a circular jacuzzi (i.e. the shape is a right circular cylinder) to hold $1000\pi\ ft^3$ of water. Find the optimum dimensions (i.e. radius and height) that will minimize the surface area. Justify that you have indeed found a minimum. What is the total surface area?

6. (30 points) For this problem you must show all work to receive full credit. Consider the function:

   $f(x) = \dfrac{x}{\ln x}$ for $x > 0$ $\qquad f'(x) = \dfrac{\ln x - 1}{(\ln x)^2}$ $\qquad f''(x) = \dfrac{2 - \ln x}{x(\ln x)^3}$

   (a) Find all vertical and horizontal asymptotes for this function. Justify your answer by considering the appropriate limits. If there are no vertical or horizontal asymptotes, please state this.

   (b) Find the x-coordinates of all critical points. Identify any relative maxima and minima. Be sure to say which is which and if none exist, say so.

   (c) Find the x-coordinates of the inflection points if any exist. Justify your answer.

   (d) Sketch a graph of $f(x)$, labeling the points you found in parts $a - c$ as appropriate.

JUST ONE MORE PAGE!!

7. (30 points) Suppose that the position at time t (seconds) of a particle moving along a coordinate axis is $s(t) = \int_0^t f(x)\,dx$ meters, where f is the differentiable function graphed below and $0 \le t \le 8$.

(a) What is the particle's velocity at time $t = 4$?

(b) Is the acceleration of the particle at time $t = 4$ positive, negative, or zero? Why?

(c) Estimate the particle's position at time $t = 5$. Justify your answer

(d) At what time during the first 8 seconds does s have its largest value?

(e) Approximately when is the acceleration zero? How do you know?

(f) When is the particle moving towards the origin?

Extra Credit (up to 5 points) In the movie *The Wizard of Oz*, when the Scarecrow receives his "brains" from the wizard, he says "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." It turns out that the Scarecrow was incorrect. What is wrong with his statement?

The following may be useful:

$$\frac{d[\sin^{-1} u]}{dx} = \frac{u'}{\sqrt{1-u^2}} \qquad |u| < 1$$

$$\frac{d[\cos^{-1} u]}{dx} = \frac{-u'}{\sqrt{1-u^2}} \qquad |u| < 1$$

$$\frac{d[\tan^{-1} u]}{dx} = \frac{u'}{1+u^2}$$

$$\frac{d[\cot^{-1} u]}{dx} = \frac{-u'}{1+u^2}$$

$$\frac{d[\sec^{-1} u]}{dx} = \frac{u'}{|u|\sqrt{u^2-1}} \qquad |u| > 1$$

$$\frac{d[\csc^{-1} u]}{dx} = \frac{-u'}{|u|\sqrt{u^2-1}} \qquad |u| > 1$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad u^2 < a^2$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad u^2 > a^2$$

$$V_{right\ cylinder} = \pi r^2 h$$

$SA_{right\ cylinder}$ = (circumference of base)(height) + area of the bases