APPM 1350 EXAM #3 Fall 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets and calculators are NOT permitted.

1. (25 points) In this problem you will calculate two different estimates for the value of $\sqrt[3]{10}$. One estimate is based on linearization of a function, and the second estimate is based on root finding using Newton's method. (Please leave your answers for parts (b) and (d) as fractions.)

   (a) Find the linearization of $g(x) = \sqrt[3]{x}$ near $x = 8$.

   (b) Use the linearization of $g(x)$ from part (a) to estimate the value of $\sqrt[3]{10}$.

   (c) Now, consider the function $f(x) = x^3 - 10$. Use Newton's Method to write down an iteration formula ($x_{n+1} = \ldots$) in terms of x_n that can be used to find the solution of $f(x) = 0$.

   (d) Starting with an initial guess of $x_0 = 1$, use the iteration formula from part (c) to calculate x_1 and x_2. Report your value of x_2 as an estimate of $\sqrt[3]{10}$.

2. (25 points) Consider the integral $\displaystyle\int_0^2 x^2\,dx$.

   (a) Estimate the value of the integral by computing its Riemann Sum with 4 equally-spaced subintervals. Use the left-hand endpoint of each subinterval to compute the height of the corresponding rectangle.

   (b) Use the Trapezoidal Rule with 4 equally-spaced subintervals to estimate the value of the integral.

   (c) Estimate the number of subintervals required to keep the magnitude of the error associated with the Trapezoidal Rule approximation less than 3×10^{-4}. (Hint: n should be rounded up to an integer value.)

3. (25 points) Evaluate the following integrals.

   (a) $\displaystyle\int \left(\sqrt[3]{x^2} - \frac{1}{x^3} + 8\right) dx$

   (b) $\displaystyle\int (5 + \sin^2\theta)\,d\theta$

   (c) $\displaystyle\int_{\sqrt{5}}^{\sqrt{12}} t\sqrt{4+t^2}\,dt$

   (d) $\displaystyle\int y^3\sqrt{4+y^2}\,dy$

4. (25 points) In this problem you will demonstrate that the function $y(x) = \dfrac{x}{2}\displaystyle\int_{\sqrt{\pi}}^{x} \sin(t^2)\,dt$ satisfies the differential equation $\dfrac{d^2y}{dx^2} = \sin(x^2) + x^2\cos(x^2)$, where $y(\sqrt{\pi}) = 0$ and $y'(\sqrt{\pi}) = 0$. The following steps will walk you through the process.

   (a) Using the proposed solution for $y(x)$ provided above, calculate $\dfrac{dy}{dx}$.

   (b) Based on your result from part (a), calculate $\dfrac{d^2y}{dx^2}$. Does your expression for $\dfrac{d^2y}{dx^2}$ satisfy the differential equation above?

   (c) Verify that the proposed solution for $y(x)$ satisfies $y(\sqrt{\pi}) = 0$. Show all your work.

   (d) Verify that the proposed solution for $y(x)$ satisfies $y'(\sqrt{\pi}) = 0$. Show all your work.