ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets and calculators are NOT permitted.

1. (21 points) Evaluate the following limits.

   (a) $\lim_{h\to 0} \dfrac{\tan(x+h)-\tan(x)}{h}$ (b) $\lim_{x\to 0^+} (\ln(x) - \ln(\sin x))$ (c) $\lim_{y\to\infty} \left(1-\dfrac{5}{y}\right)^y$

2. (21 points) Evaluate $\dfrac{dy}{dx}$ for the following expressions.

   (a) $x^2y + e^{x+y} = \sec^2(\pi)$

   (b) $y = x^{\ln x}$

   (c) $y(x) = x\displaystyle\int_\pi^x \frac{\sin(t)}{t}\,dt$
   Hint: do not try to integrate this.

3. (28 points) Evaluate the following integrals.

   (a) $\displaystyle\int y^3\sqrt{4+y^2}\,dy$

   (b) $\displaystyle\int_0^{2\pi} |\sin\theta|\,d\theta$

   (c) $\displaystyle\int_1^e \frac{(\log_5 x)^2(\log_7 x)}{x}\,dx$

   (d) $\displaystyle\int \frac{1}{x^2-2x+5}dx$

4. (20 points) Consider the function $f(x) = \sin^{-1}(x) + \cos^{-1}(x)$ for $-1 < x < 1$.

   (a) Calculate $f(0)$.

   (b) Calculate $f'(x)$.

   (c) Based on your computation in part (b), what can you conclude about f?

   (d) Use your conclusions from parts (a) and (c) to determine an identity involving $\sin^{-1}(x)$ and $\cos^{-1}(x)$.

5. (20 points) The sum of two non-negative numbers, x and y, is 3. Find the values of x and y that maximize $f(x,y) = x + y^2$.

THERE IS MORE ON THE BACK

6. (20 points) Consider the function $y = x^{5/3} - 5x^{2/3}$ for which $y' = \dfrac{5(x-2)}{x^{1/3}}$ and $y'' = \dfrac{10(x+1)}{9x^{4/3}}$.

   (a) Determine the intervals on which y is increasing, and decreasing.

   (b) Determine the intervals on which y is concave up, and concave down.

   (c) Determine the location (and value) of any local extreme values of y.

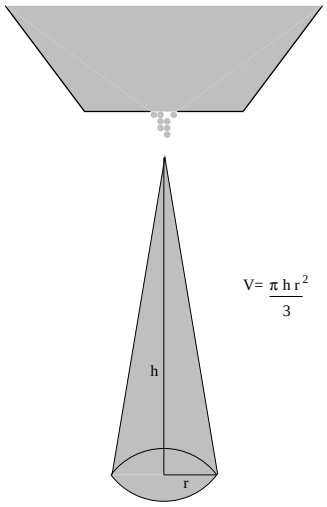
   (d) Determine the location (and value) of any inflection points.

   (e) Draw an accurate sketch of the graph labeling all critical points and inflection points. To help you, here are some points that are on the graph — only show those that are critical points when sketching the graph: $(-1,-6)$, $(-8,-62)$, $(0,0)$, $(1,-4)$, $(2,-4.8)$ and $(5,0)$.

7. (20 points) Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. (Hint: the volume of a cone of height h and base radius r is $V = \dfrac{\pi}{3} r^2 h$.)

   (a) How fast is the height changing when the pile is 4 m high?

   (b) How fast is the radius of the base changing when the pile is 4 m height?



The following formulas may be useful:

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C = -\cos^{-1}\left(\frac{u}{a}\right) + C \quad \text{for} \quad u^2 < a^2$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C = -\frac{1}{a}\cot^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C = -\frac{1}{a}\csc^{-1}\left|\frac{u}{a}\right| + C \quad \text{for} \quad u^2 > a^2$$