APPM 1350 Spring 2005 Exam1 Solution

1. (a) $\lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1} = \text{"}\dfrac{0}{0}\text{"} = \lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}\dfrac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \lim_{x\to 1}\dfrac{(x+3)-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1}\dfrac{x-1}{(x-1)(\sqrt{x+3}+2)} =$
$\lim_{x\to 1}\dfrac{1}{(\sqrt{x+3}+2)} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$
□

(b) $\lim_{x\to 2^-}\dfrac{|x-2|}{x-2} = \text{"}\dfrac{0}{0}\text{"} = \lim_{x\to 2^-}\dfrac{-(x-2)}{x-2} = \lim_{x\to 2^-} -1 = -1$
□

(c) $\lim_{x\to -5^-}\dfrac{3x}{2x+10} = \text{"}\dfrac{-15}{0}\text{"} = \text{"}\dfrac{-15}{-0.0...}\text{"} = +\infty$
□

(d) $-1 \le \cos(\frac{1}{x^2}) \le 1 \Rightarrow -x \le x\cos(\frac{1}{x^2}) \le x$ (because $x > 0$).

Since $-x \to 0$ as $x \to 0$ and $x \to 0$ as $x \to 0$, by the Sand. Th., $\lim_{x\to 0^+} x\cos(\dfrac{1}{x^2}) = 0$

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2. (a) $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$
□

(b) $\lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{1}{(x+h)-1} - \dfrac{1}{x-1}\right) = \lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{(x-1)-(x+h-1)}{(x+h-1)(x-1)}\right) = \lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{(x-1)-(x+h-1)}{(x+h-1)(x-1)}\right) =$
$= \lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{x-1-x-h+1}{(x+h-1)(x-1)}\right) = \lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{-h}{(x+h-1)(x-1)}\right) =$
$= \lim_{h\to 0}\dfrac{-1}{(x+h-1)(x-1)} = \dfrac{-1}{(x-1)(x-1)} = \dfrac{-1}{(x-1)^2}$
□

(c) equation of the line: $y = mx + b$
$m = \left.\dfrac{df(x)}{dx}\right|_{x=2} = \left.\dfrac{-1}{(x-1)^2}\right|_{x=2} = \dfrac{-1}{(2-1)^2} = \dfrac{-1}{1} = -1$
when $x = 2$, $y = f(2) = \frac{1}{2-1} = 1$
so, $1 = (-1)(2) + b \Rightarrow 1 = -2 + b \Rightarrow b = 3$
therefore, tangent line is: $y = -x + 3$

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3. (a) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 + x = 2$ AND $\lim_{x\to 1^-} f(x) = \lim_{x\to 1} 2x = 2 \Rightarrow \lim_{x\to 1} f(x) = 2$
$f(1) = (2)(1) = 2$
since $\lim_{x\to 1} f(x) = f(1) \Rightarrow$YES
□

(b) $\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+}\frac{(1+h)^2+(1+h)-2}{h} = \lim_{h\to 0^+}\frac{1+2h+h^2+1+h-2}{h} = \lim_{h\to 0^+}\frac{3h+h^2}{h} = \lim_{h\to 0^+} 3+h = 3$
$\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-}\frac{2(1+h)-2}{h} = \lim_{h\to 0^-}\frac{2+2h-2}{h} = \lim_{h\to 0^-}\frac{2h}{h} = \lim_{h\to 0^-} 2 = 2$
since $3 \neq 2$ (or since left-hand limit is different from right-hand limit)$\Rightarrow$NO, $\lim_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$ DNE
□

(c) NO, because as seen in part(b), $\left.\dfrac{df(x)}{dx}\right|_{x=1}$ DNE

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4. (a) $f(-x) = (-x)^5 + \sin(-x) + 3 = -x^5 - \sin(x) + 3 \Rightarrow$NEITHER
(b) $g(-x) = (-x)^2 + \cos(-x) + 3 = x^2 + \cos(x) + 3 = g(x) \Rightarrow$EVEN
(c) Since $f(x)$ is continuous, $f(-\pi) < 0$ and $f(\pi) > 0$, by the IVT there exists a real solution$\Rightarrow$YES
(d) Since $x^2 \ge 0$ and $\cos(x) \ge -1 \Rightarrow g(x) = x^2 + \cos(x) + 3 \ge 0 - 1 + 3 = 2 \Rightarrow g(x) \ge 2 \Rightarrow$NO