Final Exam: APPM 1350 - Spring 2005.

ON THE FRONT OF YOUR BLUEBOOK please write: (1) your name, (2) student ID, (3) section and lecturer name (010-Carvalho or 020-Lladser). You must work all the problems on the exam. Show all your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are not permitted. **START EACH PROBLEM IN A NEW PAGE**.

P1. (24 points) Determine the following limits.

(a) $\lim_{x\to 0} \dfrac{\cos(\pi x/2)}{\cos(\pi x/4)}$.

(b) $\lim_{x\to 1^+} x^{1/(x-1)}$.

(c) $\lim_{x\to\infty} \sqrt{x+1}-\sqrt{x}$.

P2. (24 points) Evaluate the following integrals. Simplify your answer to all definite integrals.

(a) $\displaystyle\int_4^9 \frac{\log_3(x)}{\log_2(x)}dx$.

(b) $\displaystyle\int \frac{1}{\sin^{-1}(y)\sqrt{1-y^2}}dy$.

(c) $\displaystyle\int \frac{1}{1+(3z+1)^2}dz$.

P3. (18 points) Consider the function

$$f(x) = \begin{cases} \sin^{-1}(x) & , \quad -1 \le x < \frac{1}{\sqrt{2}} \\ \cos^{-1}(x) & , \quad \frac{1}{\sqrt{2}} \le x \le 1 \end{cases} .$$

(a) Determine $f\left(\frac{1}{\sqrt{2}}\right)$. **Hint:** $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

(b) Is f continuous at $x = \frac{1}{\sqrt{2}}$? Explain.

(c) Is f differentiable at $x = \frac{1}{\sqrt{2}}$? Explain.

P4. (12 points) Consider the function

$$g(x) = x - 1 + \frac{\sin(x^2)}{x} .$$

(a) Does the graph of $y = g(x)$ resemble the one of a line as $x \to \infty$? If so, what is it?

(b) Does the graph of $y = g(x)$ have vertical asymptotes? If so, what are they?

(FORMULA SHEET ON THE BACK)

P5. (16 points) Consider the function

$$y(x) = \left(1 + e^{-x}\right) \cdot x^{x}\,.$$

(a) Compute $\dfrac{dy}{dx}$.

(b) Use (a) to show that $y(x)$ is a solution of the differential equation:

$$\frac{dy}{dx} - \ln(x) \cdot y = x^{x}\,.$$

P6. (16 points) The coordinates of a particle moving in the xy-plane are differentiable functions of time t with $\dfrac{dx}{dt} = -1$ m/sec and $\dfrac{dy}{dt} = -5$ m/sec. How fast is the particle approaching the origin as it passes through the point $(5, 12)$?

P7. (16 points) In this problem you will estimate the value of $\ln(3)$ using the Trapezoidal Rule to approximate definite integrals.

(a) State the definition of $\ln(x)$ in terms of a definite integral.

(b) Apply the Trapezoidal Rule with 4 equally-spaced subintervals to estimate $\ln(3)$ using the definition of the natural logarithm that you stated in part (a). Leave your final answer as a simplified fraction.

P8. (24 points) Consider the function $g(x) = \displaystyle\int_1^x f(t)\, dt$, where $f(t)$ is such that

- $f(2) = 0$, and
- $f'(t) > 0$, for all t.

Determine whether the following statements are TRUE (i.e. it is always true), FALSE (i.e. it is always false) or there is NOT ENOUGH INFO (i.e. it is sometimes true, sometimes false, depending on additional conditions). You do not need to justify your answers.

(a) $g(0)$ is less than zero.

(b) $g(2)$ is less than zero.

(c) $g(x)$ is a differentiable function of x.

(d) The graph of $g(x)$ has a horizontal tangent at $x = 2$.

(e) $g(x)$ is a twice-differentiable function of x.

(f) $g(x)$ has a local maximum at $x = 2$.

(g) The graph of $g(x)$ has an inflection point at $x = 2$.

(h) $g(x)$ is a one-to-one function of x.

Formula Sheet

$$\begin{aligned}
\sin^{-1} &: \quad \text{domain}=[-1,1];\ \text{range}=[-\pi/2,\pi/2]\,, \\
\cos^{-1} &: \quad \text{domain}=[-1,1];\ \text{range}=[0,\pi]\,, \\
\tan^{-1} &: \quad \text{domain}=(-\infty,\infty);\ \text{range}=(-\pi/2,\pi/2)\,, \\
\sec^{-1} &: \quad \text{domain}=(-\infty,-1]\cup[1,\infty);\ \text{range}=[0,\pi/2)\cup(\pi/2,\pi]\,, \\
\int \frac{1}{\sqrt{a^2-x^2}}dx &= \sin^{-1}\left(\frac{x}{a}\right)+C\,, \\
&= C-\cos^{-1}\left(\frac{x}{a}\right)\,, \\
\int \frac{1}{a^2+x^2}dx &= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C\,, \\
\int \frac{1}{x\sqrt{x^2-a^2}}dx &= \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|+C\,.
\end{aligned}$$