

On the front of your bluebook print (1) your name, (2) your student ID number and (3) a grading table. Show all your work in your bluebook and box in your final answers where appropriate. A correct answer with no supporting work may receive no credit, while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. Please start each of the five problems on a new page.

1. (15 points) Answer the following Always True or False. You do not need to show your work.

- (a) The following equation describes a circle with radius 2 and center  $(-1, 1)$ :  $x^2 + y^2 - 2x + 2y = 2$ .
- (b) The function  $y = 2x^2 + 4 \cos(x)$  is even.
- (c) If  $|-2x + 9| < 5$  then  $2 < x < 7$ .
- (d) If  $\lim_{x \rightarrow c} f(x) = f(c)$  then  $f(x)$  is continuous at  $x = c$ .
- (e) We say  $f(x)$  has a left-hand limit  $L$  if for all  $\epsilon > 0$  there exists a  $\delta > 0$  such that for all  $x$

$$0 < x - x_0 < \delta \Rightarrow |f(x) - L| < \epsilon.$$

2. (28 points) Calculate the following limits if they exist. If the limit doesn't exist, state this and explain why.

- (a)  $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$
- (b)  $\lim_{x \rightarrow 1^-} \frac{2x-4}{x-1}$
- (c)  $\lim_{x \rightarrow 2} \frac{x|x-2|}{x-2}$
- (d)  $\lim_{x \rightarrow 0} f(x) = \begin{cases} 2x^2 & \text{if } x < 0 \\ \tan(x) & \text{if } 0 \leq x \leq \frac{\pi}{2} \end{cases}$

3. (20 points) Find the interval(s) over which the following functions are continuous. If they have discontinuities, state the type of discontinuity. If possible, define the continuous extension of the function.

- (a)  $f(x) = \frac{1}{x}$
- (b)  $f(x) = \frac{x^2-9}{x+3}$
- (c)  $f(\theta) = \cos(\theta) \tan(\theta)$  on  $[-\pi, \pi]$
- (d)  $f(x) = \frac{x^2+2x}{x(x-1)}$

4. (15 points) Find a value of  $a$  such that

$$f(x) = \begin{cases} -\frac{2}{3}x + 4 & x < 3 \\ ax - 1 & x \geq 3 \end{cases}$$

is continuous at every  $x$ . Graph the function using the value of  $a$  you found. Is the function differentiable at  $x = 3$ ? Why or why not?

5. (22 points) Find the following information.

- (a) State the definition of the derivative.
- (b) Using the definition, calculate the derivative of  $y(x) = \frac{1}{\sqrt{x-2}}$ .
- (c) Find the equation of the line tangent to the curve at  $x = 6$ .