

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.

- (a) A function f has a local minimum at $x = c$ if $f'(c) = 0$ and $f''(c) < 0$.
- (b) If f is a continuous function on $[a,b]$, then f is integrable on $[a,b]$.
- (c) If f is a continuous function, then f is a differentiable function.
- (d) The function $f(x) = x^3$ has an inverse.
- (e) As $x \rightarrow \infty$, the function $3x^5 + 2x - 7$ grows at the same rate as x^5 .

2. (25 points) Evaluate the following integrals.

(a) $\int_e^{e^2} \frac{dx}{x \ln(x)}$ (b) $\int \frac{e^x dx}{1 + 9e^{2x}}$ (c) $\int_1^2 \frac{8 dx}{x^2 - 2x + 2}$

3. (25 points) Evaluate the following limits if they exist. If a limit fails to exist, or is infinite, please indicate this.

(a) $\lim_{x \rightarrow 0} \frac{\tan^2(x)}{2x^2}$ (b) $\lim_{x \rightarrow \infty} x^{(3/\ln x)}$ (c) $\lim_{h \rightarrow 0} \frac{\ln(e^2 + h) - \ln(e^2)}{h}$
(Hint: think derivative!)

4. (25 points) Calculate $\frac{dy}{dx}$ for each of the following expressions.

(a) $y^2 6^y = \cos(xy)$ (b) $y = \int_{\log_{10}(x)}^{\pi} (t^2 + 1) dt$ (c) $y = x^x$

5. (20 points) In this problem, you will verify that the derivative of $f(x) = e^x$ is $f'(x) = e^x$ by using the limit definition of the derivative.

- (a) First, show that $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$.
- (b) Next, state the definition of the derivative of $f(x)$.
- (c) Now, use your result from part (a), and the definition of the derivative from part (b), to show that $\frac{d e^x}{dx} = e^x$.

6. (25 points) In constructing a new Rec Center, a university wants to have a room consisting of a rectangular region with a semicircular section on each end. If the perimeter of the room is to be a 200-meter running track, find the dimensions of the rectangular region that will make the area of the rectangular region as large as possible. Verify that your values lead to a maximum.

THERE IS MORE ON THE BACK

7. (20 points) John Napier (1550–1617), the Scottish laird who invented logarithms, was the first person to answer the question, “What happens if you invest an amount of money A_0 at 100% interest, compounded continuously, for t years?” In this problem, please leave all of your answers in terms of logs and exponentials.
- Write the expression for the amount of money you have after t years, $A(t)$, as a function of time t , the initial amount of money, A_0 , and the interest rate, r .
 - How long does it take to triple your money (with 100% interest)?
 - What interest rate would be required to increase the amount of money in your account by a factor of eight over a period of 3 years?
8. (20 points) Consider the following questions concerning Newton’s method.
- If $f(x) = x^3 + x + 1$ and $x_0 = 0$, use Newton’s method to approximate the zero/root of $f(x)$ by calculating x_1 and x_2 .
 - Copy the following graph of the function into your exam book and on it graphically indicate what the calculations in part (a) represent. Add appropriate labels to your figure.
 - Explain the relationship between the linearization of $f(x) = x^3 + x + 1$ and your value x_1 .
9. (25 points) To find the height, H , of a lamppost, you stand a 6-ft pole 20 ft from the lamp and measure the length of its shadow, S . You measure S to be 15 ft.
- Calculate the height of the lamppost (in feet).
 - Suppose your measurement of the shadow was in error by one inch. Estimate (using Calculus) the magnitude of the resulting error (in inches) in your calculation of H .
10. (10 extra credit points) Derive the derivative of $f(x) = \arcsin x$ also written as $f(x) = \sin^{-1}(x)$.

The following formulas may be useful:

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \text{ for } u^2 < a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C \text{ for } u^2 > a^2$$