

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (10 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.
 - (a) There exists a continuous and differentiable function f such that $f(1) = -2$, $f(3) = 3$ and $f'(x) \leq 2$ for all x .
 - (b) The point $(2, -16)$ is an inflection point of $y = x^4 - 4x^3$.
 - (c) If f is a differentiable function of x , then $\frac{d}{dx}f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$.
 - (d) If $f(x)$ is continuous at $x = a$, then $f(x)$ is differentiable at a .
 - (e) If $g(x) = x^5$, then $\lim_{x \rightarrow 2} \frac{g(x) - g(2)}{x - 2} = 80$.

2. (30 points) Compute the following, if they exist. If a limit does not exist, clearly state this.
 - (a) $\lim_{t \rightarrow 0} \frac{t^2}{\tan^2(2t)}$
 - (b) $\lim_{x \rightarrow \infty} \frac{x^4 + 3x \cos x}{6 - 6x^4 - 2 \cos(x^2)}$
 - (c) $\lim_{x \rightarrow -\infty} \frac{7x^{-2} + 3x^{-3}}{5x^{-3} + 2x^{-2}}$
 - (d) $\frac{dy}{dx}$ if $y = \sin(x \cos x)$
 - (e) Find the equation of the line tangent to the curve $y(y^2 - 1)(y - 2) = x(x - 1)(x - 2)$ at the point $(0, 1)$.

3. (15 points) Use differentials to estimate the allowable percentage error in measuring the radius r of a sphere if volume is to be calculated correctly within 6%. Note: $V = \frac{4}{3}\pi r^3$.

4. (25 points) Consider the function $f(x) = \frac{-x^2}{x+1}$ for $x \neq -1$, which has the first derivative $f'(x) = \frac{-x(x+2)}{(x+1)^2}$ and second derivative $f''(x) = \frac{-2}{(x+1)^3}$.
 - (a) Locate any roots (or zeroes) of f .
 - (b) Locate and classify all extrema of f .
 - (c) Locate any inflection points of f .
 - (d) Identify all asymptotes and dominant terms of f .
 - (e) On what interval(s) is f increasing? Decreasing?
 - (f) On what interval(s) is f concave up? Concave down?
 - (g) Sketch the graph of $f(x)$ and clearly identify any roots, extreme values and inflection points. Include the graphs and equations of all asymptotes and dominant terms.

THERE IS MORE ON THE BACK

5. (20 points) A woman starts walking north at 2 miles/hour from a point P. 30 minutes later, a man starts walking south at 1 mile/hour from a point 3 miles due east of P. At what rate are the two people moving apart one hour after the man starts walking?