APPM 1350 **EXAM 3** **SPRING 2006**

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Note that a short table of formulas that may be helpful can be found on the back of this exam.

1. (10 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.
   (a) There exists no function $f(x)$ such that $f(x) = f^{-1}(x)$.
   (b) For some c in $[a, b]$, $\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$.
   (c) If $f'(x) = g'(x)$ on the interval $[a, b]$, then $f(b) - f(a) = g(b) - g(a)$.
   (d) $6 \le \int_0^3 \sqrt{4+x^2}\,dx \le 3\sqrt{13}$.
   (e) $\int_{-1}^{2} \frac{1}{x^2}\,dx = \int_{-1}^{2} x^{-2}\,dx = -\frac{3}{2}$ by the Fundamental Theorem of Calculus.

2. (25 points) Evaluate the following.
   (a) $\int \sin^2(t)\,dt$
   (b) $\int (3x^2 - 2x + 6 - \frac{1}{x^2})\,dx$
   (c) $\int x^2\sqrt{1-x}\,dx$
   (d) $\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$
   (e) Find the average value of $f(x) = 3x^2 - 2x$ on the interval $[1, 4]$

3. (25 points) Consider the integral $I = \int_0^4 \left(x^2 + 1\right)\,dx$. Suppose we want to estimate the value of I by using numerical integration.
   (a) Estimate I by using the trapezoid rule with $n = 4$ subintervals.
   (b) Find the upper bound for the error of the trapezoid rule estimation of I with $n = 4$ subintervals.

4. (20 points) Consider the function $f(x) = x^2 - 8$. Recall that Newton's method can be used to estimate the value of a root/zero of a function.
   (a) Show that the Newton's method iteration with $f(x) = x^2 - 8$ can be written as $x_{n+1} = \frac{1}{2}(x_n + \frac{8}{x_n})$.
   (b) Use the iterative formula of part a) to estimate the value of $+\sqrt{8}$, by using $x_0 = 4$ to find x_1 and x_2.
   (c) What values for x_0 could be used in part b) to estimate the value of $+\sqrt{8}$ using Newton's method?

THERE IS MORE ON THE BACK

5. (20 points) Many special functions in mathematics, physics and engineering are defined as integrals. One such example that arises in optics is the *Fresnel sine function*: $S(x) = \sqrt{\frac{2}{\pi}} \int_0^x \sin(t^2)\, dt$.

(a) Find $S'(x)$.

(b) Find $S''(x)$.

(c) Show that $S(x)$ is a solution to the initial value problem $[S'(x)]^2 + \left(\frac{S''(x)}{2x}\right)^2 = \frac{2}{\pi}$, $S(0) = 0$.

(d) Find any local maximum or minimum values of $S(x)$ on the interval $[0, \sqrt{2\pi}]$. Also find any inflection points of $S(x)$ on the same interval.

Some formulas that may be helpful:

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$|E_T| \leq \frac{b-a}{12} h^2 M$$

$$E = mc^2$$