

1. (25 points, 5 points each)

$$(a) g'(y) = (y) \left(\cos \frac{1}{y} \right) \left(\frac{-1}{y^2} \right) + \sin \frac{1}{y}$$

$$(b) s'(r) = (2 \tan r^3)(\sec^2 r^3)(3r^2)$$

$$(c) f'(x) = \frac{1}{2\sqrt{1 + \tan\left(x + \frac{1}{x}\right)}} \left[\sec^2\left(x + \frac{1}{x}\right) \right] \left(1 - \frac{1}{x^2}\right)$$

$$(d) p'(t) = 2 \left[\left(1 + \frac{2}{t}\right)^{-1} + 3t \right] \left[-\left(1 + \frac{2}{t}\right)^{-2} \left(\frac{-2}{t^2}\right) + 3 \right]$$

$$(e) h'(x) = [\cos(\sec(\cos x))][\sec(\cos x) \tan(\cos x)](-\sin x)$$

2. (20 points, 5 points each)

$$(a) \lim_{t \rightarrow \infty} \frac{3t^{3/4} + t^{5/6} + t^2}{4t^{7/3} + \frac{2}{5}\sqrt{t}} = \lim_{t \rightarrow \infty} \frac{3t^{-19/12} + t^{-3/2} + t^{-1/3}}{4 + \frac{2}{5}t^{-11/6}} = \frac{0}{4} = 0$$

$$(b) \lim_{x \rightarrow -\infty} \frac{|x+3|}{(x+3)} = \lim_{x \rightarrow -\infty} \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases} = \lim_{x \rightarrow -\infty} -1 = -1$$

$$(c) \lim_{r \rightarrow \infty} \sqrt{r^2 + 1} - r = \lim_{r \rightarrow \infty} (\sqrt{r^2 + 1} - r) \left(\frac{\sqrt{r^2 + 1} + r}{\sqrt{r^2 + 1} + r} \right) = \lim_{r \rightarrow \infty} \frac{1}{\sqrt{r^2 + 1} + r} = 0$$

$$(d) \lim_{y \rightarrow -\infty} \frac{y^3}{y^2 + 1} = \lim_{y \rightarrow -\infty} \frac{y}{1 + \frac{1}{y^2}} = -\infty$$

Dominant term:

$$y^2 + 1 \quad \frac{y}{|y^3|} \\ \frac{-y^3 - y}{-y}$$

$$\text{so } \frac{y^3}{y^2 + 1} = y - \frac{y}{y^2 + 1} \text{ and the dominant term is } y.$$

3. (15 points) Consider the curve called the *conchoid of Nicomedes* described by the equation $x^2 y^2 = (y+1)^2(4-y^2)$

(a) Find $\frac{dy}{dx}$.

$$(x^2)(2y \frac{dy}{dx}) + (2x)(y^2) = 2(y+1)\left(\frac{dy}{dx}\right)(4-y^2) + (y+1)^2(-2y \frac{dy}{dx})$$

$$2x^2 y \frac{dy}{dx} + 2xy^2 = 2 \frac{dy}{dx} (y+1)(4-y^2) - 2y \frac{dy}{dx} (y+1)^2$$

$$\frac{dy}{dx} \cdot (2x^2 y - 2(y+1)(4-y^2) + 2y(y+1)^2) = -2xy^2$$

$$\frac{dy}{dx} = \frac{-xy^2}{x^2 y - (y+1)(4-y^2) + y(y+1)^2}$$

(b) Find the tangent to the conchoid at the point (0,-2).

$$\frac{dy}{dx} \Big|_{(0,-2)} = 0 \quad \Rightarrow \quad y = -2$$

4. (20 points) $f(x) = 2 \cos x + \sin 2x$ for $0 \leq x \leq 2\pi$

$$f'(x) = -2 \sin x + 2 \cos 2x = -2 \sin x + 2(1 - 2 \sin^2 x) = -2(2 \sin^2 x + \sin x - 1) = -2(2 \sin x - 1)(\sin x + 1)$$

Critical points:

$$\sin x = \frac{1}{2} \text{ or } \sin x = -1 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$f''(x) = -2 \cos x - 4 \sin 2x = -2 \cos x - 8 \cos x \sin x = -2 \cos x(1 + 4 \sin x)$$

Possible inflection points:

$$\cos x = 0 \text{ or } \sin x = -\frac{1}{4} \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ and at } x = \alpha_1, \alpha_2 \text{ where } \sin \alpha_n = -\frac{1}{4} \text{ and } \frac{\pi}{2} < \alpha_1 < \frac{3\pi}{2} \text{ and } \frac{3\pi}{2} < \alpha_2 < 2\pi$$

Increasing/Decreasing:

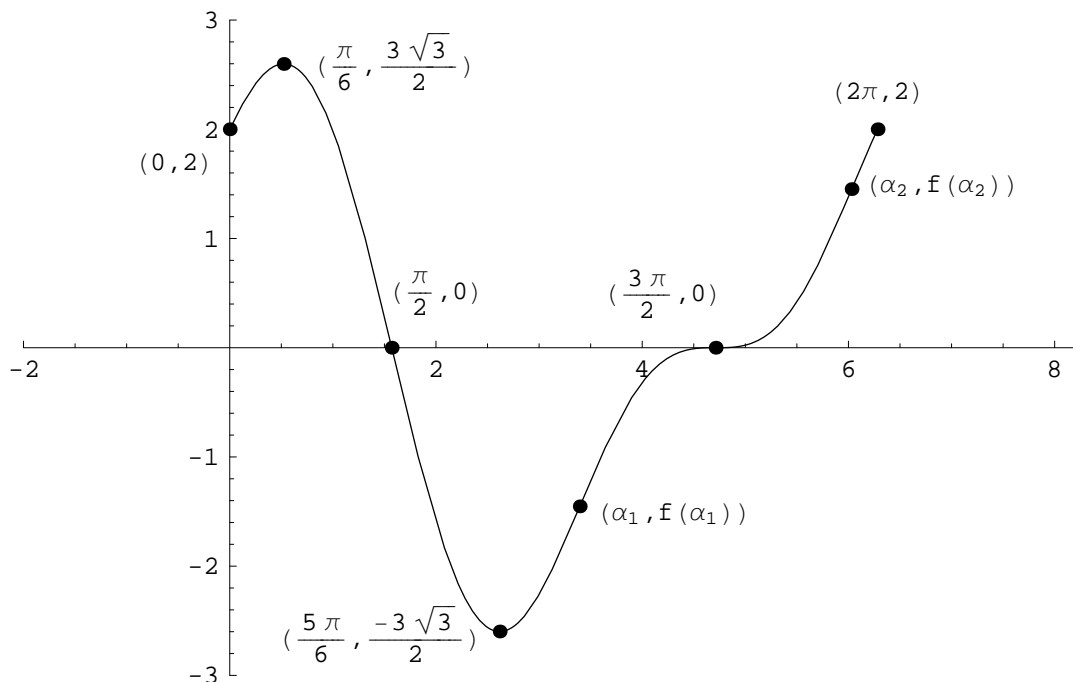
$$f'(x) \quad \left(0, \frac{\pi}{6}\right) \mid \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \mid \left(\frac{5\pi}{6}, \frac{3\pi}{2}\right) \mid \left(\frac{3\pi}{2}, 2\pi\right)$$

$$\quad \quad \quad + \quad \quad \quad - \quad \quad \quad + \quad \quad \quad +$$

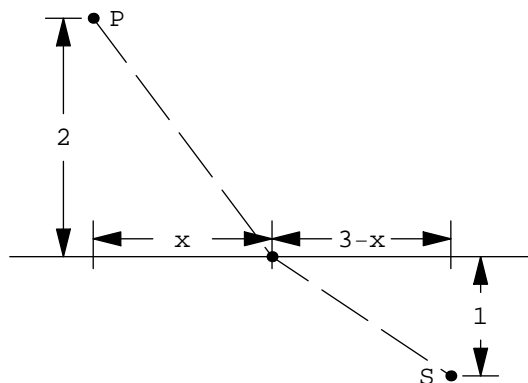
Concave Up/Down:

$$f''(x) \quad \left(0, \frac{\pi}{2}\right) \mid \left(\frac{\pi}{2}, \alpha_1\right) \mid \left(\alpha_1, \frac{3\pi}{2}\right) \mid \left(\frac{3\pi}{2}, \alpha_2\right) \mid \left(\alpha_2, 2\pi\right)$$

$$\quad \quad \quad - \quad \quad \quad + \quad \quad \quad - \quad \quad \quad + \quad \quad \quad -$$



5. (20 points)



Call the point he rows to on the shoreline Q. The distances PQ and SQ are then given by:

$$PQ = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4} \quad \text{and} \quad SQ = \sqrt{(3-x)^2 + 1}$$

The total time it takes to travel from P to Q is then:

$$\begin{aligned} T &= \frac{\sqrt{x^2 + 4}mi}{2\frac{mi}{hr}} + \frac{\sqrt{(3-x)^2 + 1}mi}{4\frac{mi}{hr}} \\ &= \frac{\sqrt{x^2 + 4}}{2} + \frac{\sqrt{(3-x)^2 + 1}}{4}hr \end{aligned}$$

Differentiate to find the shortest time:

$$\begin{aligned} T'(x) &= 2 \left(\frac{1}{2}(x^2 + 4)^{-1/2}(2x) \right) + 4 \left(\frac{1}{2}((3-x)^2 + 1)^{-1/2}(-2(3-x)) \right) \\ &= \frac{2x}{\sqrt{x^2 + 4}} - \frac{4(3-x)}{\sqrt{(3-x)^2 + 1}} = 0 \end{aligned}$$

$$\begin{aligned} \frac{x}{\sqrt{x^2 + 4}} &= \frac{3-x}{2\sqrt{(3-x)^2 + 1}} \\ \frac{x^2}{x^2 + 4} &= \frac{(3-x)^2}{4((3-x)^2 + 1)} \end{aligned}$$

$$\begin{aligned} -3(x^4 - 6x^3 + 9x^2 + 8x - 12) &= 0 \\ -3(x-1)(x^3 - 5x^2 + 4x + 12) &= 0 \\ \Rightarrow x &= 1 \end{aligned}$$

So the man must aim at a point on the shore line 1 mile downriver.