On the front of your bluebook write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. SHOW ALL YOUR WORK in your bluebook and $\boxed{\text{BOX}}$ in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. **Please start each new problem on a new page of the bluebook.**

1. (12 points) For each of the following unrelated questions, answer either ALWAYS TRUE, ALWAYS FALSE, or NEITHER. No justification is necessary.

   (a) $\left| \int_a^b f(x)\, dx \right|$ will find the area between the curve and the x-axis on the interval $[a, b]$.

   (b) $\dfrac{d}{dx} \displaystyle\int_0^x g(t^2)\, dt \; = \; 2\, x\, g(x^2)$

   (c) If $f(x)$ is odd, then $\displaystyle\int_0^x f(t)^2\, dt$ is odd.

2. (42 points) Evaluate the following expressions:

   (a) $\displaystyle\int_0^{13} \frac{1}{\sqrt[3]{(1+2x)^2}}\, dx$

   (b) $\displaystyle\int \cos^5 \frac{\theta}{2}\, d\theta$

   (c) $\displaystyle\int \csc^2 \sqrt{x} \sqrt{\frac{\tan \sqrt{x}}{x}}\, dx$

   (d) $\displaystyle\int_0^a x^3 \sqrt{a^2 - x^2}\, dx$

   (e) $\displaystyle\int \frac{(2t+1)^2}{\sqrt{t+3}}\, dt$

   (f) $\displaystyle\sum_{k=1}^{n} (2 + (k-1)^3)$

   (g) $\displaystyle\sum_{k=-2}^{3} \sin \frac{k\pi}{3} \cos k\pi$

3. (20 points) Since raindrops grow as they fall, their surface area increases and therefore their resistance to falling increases. A raindrop has an initial downward velocity of 10 m/s and its downward acceleration is given by:

$$a(t) = \begin{cases} 9 - 0.9\, t & \text{for} \quad 0 \le t \le 10 \\ 0 & \text{for} \quad t > 10 \end{cases}$$

   The raindrop is formed at 500 m above the ground.

   (a) Find the distance of the raindrop above the ground at time t.

   (b) How long does it take for the raindrop to reach the ground?

   (c) With what velocity does the raindrop strike the ground?

THE EXAM IS CONTINUED ON THE BACK

4. (12 points) If $F(x) = \int_{x-1}^{x+1} f(t)\,dt$ and $f(t) = \int_{1}^{t^2} \frac{\sqrt{w^4 - 1}}{w}\,dw$, find $F''(2)$.

5. (14 points) Find the Riemann sum for the function $f(x) = 2 + (x-2)^2$ on the interval $[0, 2]$ using 4 subintervals and the left endpoint of each subinterval. Sketch the graph of f and the approximating rectangles.

<u>Some Useful Information</u>

$$\sin A \pm B = \sin A \cos B \pm \cos A \sin B$$
$$\cos A \pm B = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos A \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$
$$\sin A \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$