
On the front of your bluebook write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. SHOW ALL YOUR WORK in your bluebook and BOX in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. **Please start each new problem on a new page of the bluebook.**

---

1. (15 points) For each of the following unrelated questions, answer either ALWAYS TRUE, ALWAYS FALSE or NEITHER. No justification is necessary.

   (a) If $f(x) > 1$ for all x and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) > 1$.

   (b) All continuous functions have derivatives.

   (c) The derivative of the function $\tan^2 x$ is the derivative of $\sec^2 x$.

   (d) $\int_{-5}^{5} (ax^2 + bx + c)\, dx = 2\int_{0}^{5} (ax^2 + c)\, dx$

   (e) If $\lim_{x\to 6} f(x)g(x)$ exists, then the limit is $f(6)g(6)$.

2. (21 points) Find $\frac{dy}{dx}$ in each case. No simplification is necessary.

   (a) $y = x \ln(\arccos x)$

   (b) $y = x^{e^x}$

   (c) $x\, e^y = \ln xy + \arctan y$

3. (28 points) Evaluate each of the following limits, if it exists. If the limit does not exist, state this and state your justification. Show all your work.

   (a) $\lim_{t\to 0^+} t^{t^2}$

   (b) $\lim_{r\to\infty} \left(re^{1/r} - r\right)$

   (c) $\lim_{x\to 0} x \,\mathrm{arccot}\frac{1}{x}$

   (d) $\lim_{h\to 0} \frac{1}{h}\int_{2}^{2+h} \sqrt{1+t^3}\, dt$

4. (21 points) Evaluate each of the following integrals.

   (a) $\int \frac{e^{-x}}{1+e^{-2x}}\, dx$

   (b) $\int \tan x \ln(\cos x)\, dx$

   (c) $\int_0^2 \frac{t}{9-t^2}\, dt$

5. (25 points) Do ONE of the following two problems. State clearly which problem you have chosen in your bluebook. Only work for the chosen problem will be graded.

   (a) What is the area of the largest rectangle in the first quadrant with two sides on the axes and one vertex on the curve $y = e^{-x}$?

   (b) Plot the function $f(x) = x\ln x$ on $(0, e]$. Find and label all critical points, local and absolute extrema, and inflection points.

THE EXAM IS CONTINUED ON THE BACK

6. (40 points)

   (a) What does it mean for $f(x)$ to be continuous at $x = a$?

   (b) What does it mean for $f(x)$ to be differentiable at $x = a$?

   (c) State both parts of the Fundamental Theorem of Calculus.

   (d) If f is a continuous function such that

$$\int_0^x f(t)\,dt = x\,\sin x + \int_0^x \frac{f(t)}{1+t^2}\,dt$$

   for all x, find an explicit formula for $f(x)$.

Some Useful Information

$$\sin A \pm B = \sin A \cos B \pm \cos A \sin B$$
$$\cos A \pm B = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin A \sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos A \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$
$$\sin A \cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$$
$$\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$$
$$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$$
$$\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\operatorname{arccsc} x = -\frac{1}{|x|\sqrt{x^2-1}}$$