

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation section on the front of your bluebook. Also make a scoring table, with places for 5 problems, plus a total score. This exam has 5 problems, on both sides of this sheet. Work all **5 problems**. Start each problem on a **new page**. Show your work. Box in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points) Consider $f(x) = 2x - x^2$ on the interval $[0,3]$
- Sketch the graph of the curve $y = f(x)$.
 - What is the average value of $f(x) = 2x - x^2$ on $[0,3]$?
 - At what value or values of x in $[0,3]$ is the average value of $f(x)$ attained?

2. (20 points)
- State BOTH PARTS of the Fundamental Theorem of Calculus completely, including the hypotheses.
 - Suppose $\int_0^x f(t)dt = \sin x$. Find $f(x)$.

3. (20 points) Evaluate the following:

- $\int_1^4 \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx$
- $\frac{d}{dx} \int_1^{x^2} \sec(t-1)dt$

4. (20 points)
- Find $f^{-1}(x)$ when $f(x) = x^2 - 2x + 1$, $x \geq 1$
 - What are the domain and range of the $f^{-1}(x)$ found in part (a)?
 - Find y' when $y = \frac{\ln x}{1 + \ln x}$. Simplify your answer.

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5. (20 points) Consider $f(x) = 3x^2$ and a partition of the interval $[0,1]$ into 2 subintervals of equal length.

a) Using left hand endpoints of each subinterval to evaluate f , approximate $\int_0^1 3x^2 dx$

by the Riemann sum, $R = \sum_{k=1}^n f(c_k)\Delta x$

b) Use the Trapezoidal Rule, $T = \frac{h}{2}(y_0 + 2[y_1 + y_2 + \dots + y_{n-1}] + y_n)$ with the same two

subintervals to approximate $\int_0^1 3x^2 dx$

c) Is R or T a better estimate of $\int_0^1 3x^2 dx$? Explain.

d) What is the smallest number of subintervals of $[0,1]$ that would be enough to be sure that the trapezoidal rule for $f(x) = 3x^2$ would have an error less than or equal to 0.5×10^{-4} , given that $|E_T| \leq \frac{b-a}{12} h^2 M$?