APPM 1350, Fall 2006, Exam 1 Solutions

1a. $f'(x) = \dfrac{(x^2-1)2-(2x+1)2x}{(x^2-1)^2}$ (HW §2.2 #18)

b. $dy/dx = -6x^2 + 18x^{-7}$

c. $\left.\dfrac{d}{dx}(uv)\right|_{x=0} = v(0)u'(0) + u(0)v'(0) = (-1)(-3)+5(2) = \boxed{13}$
(HW §2.2 #39)

2. a; True; b; False; c; True; d; True; e. True. (a is HW §P.3 #62)

3a. $-x^2 \le x^2\cos\left(\frac{9}{x}\right) \le x^2$; $\lim\limits_{x\to 0} -x^2 = \lim\limits_{x\to 0} x^2 = 0$

By Sandwich Theorem $\lim\limits_{x\to 0} x^2\cos\left(\frac{9}{x}\right) = \boxed{0}$

b. $\lim\limits_{x\to -2^+} \dfrac{(x+3)|x+2|}{x+2} = \lim\limits_{x\to -2^+} \dfrac{(x+3)\cancel{(x+2)}}{\cancel{x+2}} = \lim\limits_{x\to -2^+} x+3 = -2+3 = 1$

$\lim\limits_{x\to -2^-} \dfrac{(x+3)|x+2|}{x+2} = \lim\limits_{x\to -2^-} \dfrac{-(x+3)(x+2)}{x+2} = \lim\limits_{x\to -2^-} -(x+3) = -(-2+3) = -1$

$1 \neq -1$, Thus, $\lim\limits_{x\to -2} \dfrac{(x+3)|x+2|}{x+2}$ $\boxed{\text{DNE}}$ (HW §1.4 #17)

c. $\lim\limits_{h\to 0} \dfrac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}\left(\dfrac{\sqrt{h^2+4h+5}+\sqrt{5}}{\sqrt{h^2+4h+5}+\sqrt{5}}\right)$

$= \lim\limits_{h\to 0} \dfrac{h^2+4h+5-5}{h\left(\sqrt{h^2+4h+5}+\sqrt{5}\right)} = \lim\limits_{h\to 0} \dfrac{\cancel{h}(h+4)}{\cancel{h}\left(\sqrt{h^2+4h+5}+\sqrt{5}\right)}$

$= \dfrac{0+4}{\sqrt{0^2+4(0)+5}+\sqrt{5}} = \boxed{\dfrac{2}{\sqrt{5}}}$ (HW §1.4 #15)

d. $\lim\limits_{x\to 0} \sin\left(\frac{\pi}{2}+\tan x\right) = \sin\left(\frac{\pi}{2}+\tan(0)\right) = \sin\left(\frac{\pi}{2}+0\right) = \boxed{1}$

4 a. $y=f(x)$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ provided limit exists.

b. $f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h-1)} - \frac{1}{x-1}}{h} = \lim_{h\to 0} \frac{x-1-(x+h-1)}{h(x-1)(x+h-1)} = \frac{-1}{(x-1)^2}$

c. $-1 = \frac{-1}{(x-1)^2} \Rightarrow (x-1)^2 = 1 \Rightarrow x = 0, 2$

$f(0) = \frac{1}{0-1} = -1$ point $(0,-1)$

$f(2) = \frac{1}{2-1} = 1$ point $(2,1)$

$\boxed{\begin{array}{l} y = -1 - x \\ y = 1 - 1(x-2) \end{array}}$

(HW §1.6 #25)

5. $f(x) = \cos^2 x - x$ is continuous because of continuity of algebraic combinations of $y = \cos x$ and $y = x$; We know $\cos x$ and polynomial x are continuous.

$f(\pi/2) = -\pi/2 < 0$ and $f(-\pi/2) = \pi/2 > 0$

$\Rightarrow f$ has a root between $-\pi/2$ and $\pi/2$ by I.V.T.

6. $s(t) = -16t^2 + 96t + 56$ ft.

a. $v(t) = s'(t) = -32t + 96$ ft/sec

$a(t) = v'(t) = -32$ ft/sec^2

b. max height when $v(t) = 0$,

$\Rightarrow -32t + 96 = 0 \Rightarrow t = \boxed{3 \text{ sec}}$

c. $s(3) = -16(3^2) + 96(3) + 56$

$= \boxed{200 \text{ feet}}$