APPM 1350 - Exam 2 - Answers

1. a) $$\lim_{x\to 0}\frac{x^2-3x+\sin(x)}{4x} = \lim_{x\to 0}\left(\frac{x}{4}\right) - \lim_{x\to 0}\left(\frac{3}{4}\right) + \frac{1}{4}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$$
$$= 0 - \frac{3}{4} + \frac{1}{4} = \boxed{-\frac{1}{2}}$$

b) $$\lim_{x\to\infty}\frac{x^2-3x+\sin(x)}{4x^2+1} = \lim_{x\to\infty}\frac{\cancel{x^2}\left[1-\frac{3}{x}+\frac{\sin(x)}{x^2}\right]}{\cancel{x^2}\left[4+1/x^2\right]} = \boxed{\frac{1}{4}}$$

because $\frac{3}{x}\to 0$, $\frac{1}{x^2}\to 0$, & $\frac{\sin(x)}{x^2}\to 0$ as $x\to\infty$ $[|\sin x|\le 1]$.

c) $r(\theta)$ is defined by: $\cos(r)+\cos(\theta)=r\theta$ $\boxed{\text{HW: §2.6, \#36, p.170}}$

$\frac{d}{d\theta}[\text{eq'n}] \Rightarrow \quad -\sin(r)\frac{dr}{d\theta}-\sin(\theta) = \theta\cdot\frac{dr}{d\theta}+r$

solve for $\left(\frac{dr}{d\theta}\right) \Rightarrow$ $\boxed{\frac{dr}{d\theta} = -\frac{r+\sin(\theta)}{\theta+\sin(r)}}$

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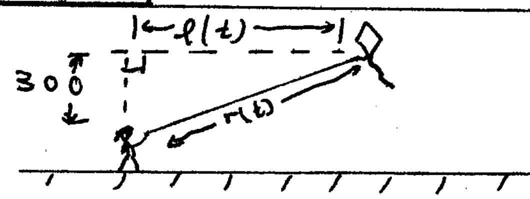
2. $\boxed{\text{HW: §2.7, \#15, p. 177}}$



Pythagorean theorem $\Rightarrow$
$$(r(t))^2 = 300^2 + (l(t))^2$$

$2r\frac{dr}{dt} = 0 + 2l\cdot\frac{dl}{dt}$ & when $r=500$, $l=400$ ft.

Problem says: $\frac{dl}{dt}=25$ ft/sec $\Rightarrow$ $2\cdot 500\cdot\frac{dr}{dt} = 2\cdot 400\cdot 25$

$\Rightarrow$ $\boxed{\frac{dr}{dt} = 20 \text{ ft/sec}}$

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3. $h(\theta) = [1+\cos(2\theta)]^{1/3}$. At $\theta=\frac{\pi}{4}$, $\cos(2\theta)=0$, $h(\frac{\pi}{4}) = 1^{1/3} = 1$.

$\frac{dh}{d\theta} = \frac{1}{3}[1+\cos(2\theta)]^{-2/3}[-\sin(2\theta)\cdot 2]$. At $\theta=\frac{\pi}{4}$, $\frac{dh}{d\theta}(\frac{\pi}{4}) = -2/3$

a) Linearization: $h(\theta)-h_0 = m\cdot(\theta-\theta_0)$

$\theta_0=\frac{\pi}{4}$, $h_0 = h(\frac{\pi}{4}) = 1$, $m = \left.\frac{dh}{d\theta}\right|_{\theta=\pi/4} = -2/3$

so $\boxed{h(\theta)-1 = -\frac{2}{3}(\theta-\pi/4)}$ This is an approximation to the correct formula for $h(\theta)$, valid for small $(\theta-\pi/4)$.

or $\boxed{L(\theta) = 1-2/3(\theta-\pi/4)}$

b) At $\theta = 0.3\pi = \frac{3\pi}{10}$, $h(0.3\pi) = 1-\frac{2}{3}\left(\frac{3\pi}{10}-\frac{\pi}{4}\right) = 1-\frac{2}{3}\left(\frac{\pi}{20}\right)$

$$\boxed{h(0.3\pi) = 1-\frac{\pi}{30}}$$

$\boxed{4}$ $f(x) = x + \frac{1}{x+1}$, $x \neq -1$

a) find maxima & minima

$f'(x) = 1 - \frac{1}{(x+1)^2}$. $f'(x)$ is defined everywhere except at $x = -1$

$f'(x) = 0 = 1 - \frac{1}{(x+1)^2} \Rightarrow (x+1)^2 = 1$

$\Rightarrow \underline{x = 0}$ or $\underline{x = -2}$

$f''(x) = \frac{2}{(x+1)^3}$

At $\underline{x=0}$, $f(0) = 1$, $f''(0) = 2 > 0$

$\boxed{x = 0, f = 1}$ is a $\underline{\text{minimum}}$

At $x = -2$, $f(-2) = -3$, $f''(-2) = -2 < 0$

$\boxed{x = -2}\,\boxed{f = -3}$ is a $\underline{\text{maximum}}$

b) Asymptotes:

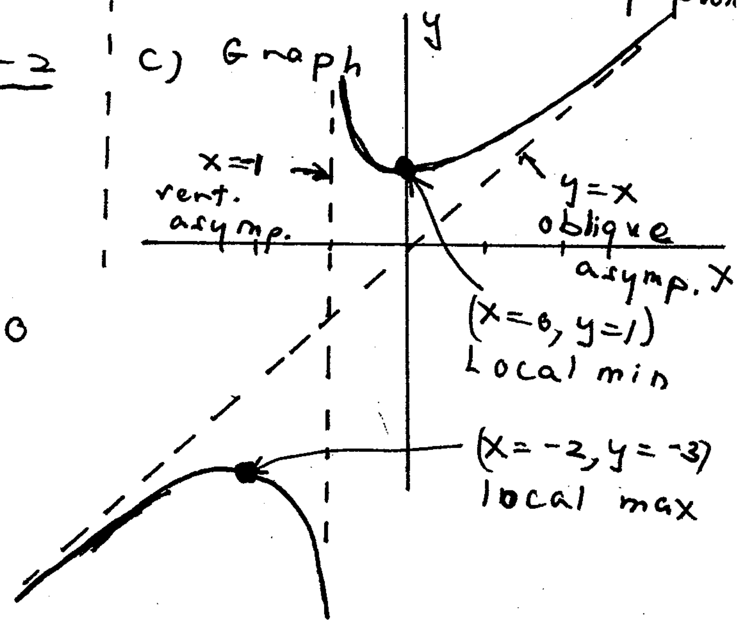
① $\boxed{x = -1}$ is a vertical asymptote

② As $x \to \pm\infty$, $f(x) \to x$

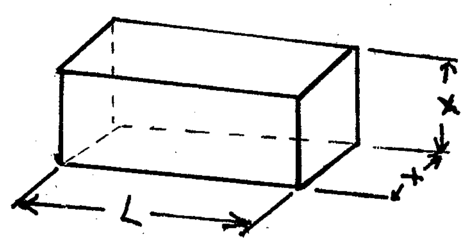
$\Rightarrow \boxed{y = x}$ is an oblique asymptote

No horizontal asymptote

c) Graph



$\boxed{5}$ $\boxed{\text{HW: § 3.6, \# 19, p. 243}}$



Volume of box

$V = Lx^2$

Girth = $4x$

Constraint: $4x + L \leq 108$. At the maximum, $4x + L = 108$

$\Rightarrow L = 108 - 4x$

$\Rightarrow V = (108 - 4x) \cdot x^2$

$\frac{dV}{dx} = 2 \cdot 108x - 12x^2 = 0$ at critical point

$\Rightarrow 12x(18 - x) = 0$

$x = 0 \Rightarrow V = 0$ minimum volume

$x = 18 \Rightarrow L = 108 - 72 = 36$

$\boxed{V = 18 \cdot 18 \cdot 36 \text{ in}^3}$

Check for max at $x = 18$: $\frac{d^2V}{dx^2} = 2 \cdot 108 - 24x$

at $x = 18$, $\frac{d^2V}{dx^2} = 216 - 24 \cdot 18 = -216 < 0 \Rightarrow \underline{\underline{\text{max}}}$

$\boxed{x = 18, L = 36}$ for max volume