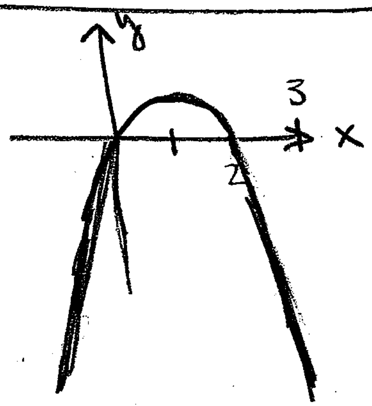
Test 3 Answer Key

1\. a)



b) $$\frac{1}{3-0}\int_0^3 (2x - x^2)\,dx = \text{AVG } F$$

$$= \frac{1}{3}\left[x^2 - \frac{x^3}{3}\right]\Bigg|_0^3 = \frac{1}{3}\left(9 - \frac{27}{3} - 0\right)$$

$$= \frac{1}{3}(0) = \boxed{0}$$

c) $$2x - x^2 = 0$$
$$x(2-x) = 0$$
$$x = 0, 2 \qquad \boxed{2 \in (0,3)}$$

2\. a) See Book p 333, 336

b) $$\int_0^x f(t)\,dt = \sin x$$

$$\Rightarrow \frac{d}{dx}\int_0^x f(t)\,dt = \frac{d}{dx}(\sin x)$$

$$\boxed{f(x) = \cos x}$$

3\. a) $$\int_1^4 \frac{dx}{\sqrt{x}\,\sqrt[4]{1+\sqrt{x}}} \qquad u = 1+\sqrt{x} \qquad u(1) = 2$$
$$du = \frac{1}{2\sqrt{x}}\,dx \qquad u(4) = 3$$

$$= 2\int_2^3 \frac{1}{u^{1/4}}\,\frac{dx}{2\sqrt{x}} = 2\int_2^3 u^{-1/4}\,du = (2)\frac{4}{3}u^{\frac{3}{4}}\Big|_2^3$$

$$= \boxed{\frac{8}{3}\left(\sqrt[4]{27} - \sqrt[4]{8}\right)}$$

b) $$\frac{d}{dx}\int_1^{x^2} \sec(t-1)\,dt \Rightarrow \boxed{[\sec(x^2-1)]\,2x} \quad \text{FTC 1}$$ HW 4.7 #78

4a) $y=(x-1)^2$

1-1 for $x \ge 1$ passes horiz line test

$x=(y-1)^2$

HW 6.1 #16

$\sqrt{x} = y-1$

choose $(+\sqrt{x}+1)$ to correspond to restricted part of the graph

$$\boxed{f^{-1}(x) = \sqrt{x}+1 \qquad x \ge 0}$$

c) $\left.\begin{aligned} D(f) &= x \ge 1 \\ R(f) &= y \ge 0 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} &D(f^{-1}): \{x \mid x \ge 0\} \\ &Rg(f^{-1}): \{y \mid y \ge 1\} \end{aligned}}$

hence choice of $f^{-1}(x) = \sqrt{x}+1$ (positive root)

d) $y = \dfrac{\ln x}{1+\ln x}$

$$y' = \frac{(1+\ln x)\frac{1}{x} - (\ln x)\left(\frac{1}{x}\right)}{(1+\ln x)^2} = \frac{\frac{1}{x} + \cancel{\frac{1}{x}\ln x} - \cancel{\frac{1}{x}\ln x}}{(1+\ln x)^2}$$

$$= \boxed{\frac{1}{x\,(1+\ln x)^2}}$$

HW 6.2 #21

5. $\Delta x = \dfrac{1-0}{2} = \dfrac{1}{2}$ $\quad P = \{0, \tfrac{1}{2}, 1\}$

a) $R = \frac{1}{2}\left(f(0) + f(\tfrac{1}{2})\right) = \frac{1}{2}\left(0 + 3(\tfrac{1}{4})\right) = \boxed{\dfrac{3}{8}}$

b) $T = \dfrac{\frac{1}{2}}{2}\left(f(0) + 2f(\tfrac{1}{2}) + f(1)\right) = \frac{1}{4}\left(0 + (\tfrac{3}{4})2 + 3\right) = \boxed{\dfrac{9}{8}}$

c) $\displaystyle\int_0^1 3x^2\,dx = x^3\Big|_0^1 = 1^3 - 0^3 = \boxed{1}$

$\boxed{\text{T is better}}$ since it is closer to actual value

d) $|E_T| \le \left|\frac{1}{12}\left(\frac{1}{n}\right)^2 6\right| \le \dfrac{1}{2 \times 10^4}$

$f(x) = 3x^2$
$f'(x) = 6x$
$f''(x) = 6 = M$

$\cancel{\tfrac{1}{2}}\,\dfrac{1}{n^2} \le \cancel{\tfrac{1}{2}} \cdot \dfrac{1}{10^4}$

$n^2 \ge 10^4$

$n \ge 100$ $\quad \boxed{n = 100}$