

*Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. Solve all 5 problems. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

1. (20 points)

(a) Calculate the limit, if it exists. If not, explain why.

$$\lim_{x \rightarrow 0} \frac{x^2 + 3x - \sin^2 x}{2x^2}$$

(b) Calculate:  $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - \sin^2 x}{2x^2 + 1}$

(c) Using implicit differentiation, find the equation of the tangent line to the cissoid of Diocles  $y^2(2 - x) = x^3$  at  $(1, 1)$ .

2. (20 points)

A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. How fast is the shadow of the ball moving along the ground 0.5 seconds later? (Assume the ball falls the distance  $s = 16t^2$  ft in  $t$  seconds.)

3. (20 points)

Define  $f(x) = \cos(\sqrt[3]{x + 1}\pi)$

(a) Calculate the linearization  $L(x)$  of  $f(x)$  about  $x = 0$ .

(b) Use  $L(x)$  from (a) to estimate  $\cos(\sqrt[3]{1.1}\pi)$ .

4. (20 points)

Consider the function  $f(x) = x - \frac{1}{x-1}$ , for  $x \neq 1$ .

(a) Find the  $x$  and  $y$  coordinates of all local maxima and minima. Identify which are maxima and which are minima. Justify your answers.

(b) Determine any horizontal or vertical asymptotes the graph of  $f$  might have.

(c) Graph  $f$ , using the information from (a) and (b). In your graph, show and label all maxima, minima and asymptotes.

5. (20 points)

(a) Show that at some instant during a 2-hour automobile trip, the speedometer reading will equal the average speed of the trip.

(b) To make a box, you use a square piece of cardboard with sidelength  $L = 10$  inches. Out of all four corners of the piece, you cut identical squares of sidelength  $x$  inches, then you fold up the edges. How large does  $x$  have to be for the box to have the largest volume?