

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. There are 5 problems of 20 points each, plus a 20-point extra credit problem. Show your work clearly and box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

(1) (a) Estimate the integral $\int_1^2 \frac{1}{x} dx$ using $n = 4$ rectangles and left-hand end-points.

(b) Estimate the same integral using the trapezoidal sum for $n = 4$.

(c) How large would you have to make n to be sure that the corresponding trapezoidal estimate is within 0.01 of the real value of the integral?

Hint: The trapezoidal sum and error bound for the trapezoidal rule are:

$$T_n = \frac{h}{2}(y_0 + 2y_1 + \dots + 2y_{n-1} + y_n)$$

$$|E_T| \leq \frac{b-a}{12} h^2 M$$

(2) Find the total area of the two regions enclosed between the graph of the function $y = 3x - x^2$ on the interval $[0, 4]$ and the x -axis.

(3) Calculate the integrals:

(a) $\int \frac{\sqrt{v} + 1}{\sqrt[3]{v}} dv$

(b) $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$

(c) $\int_0^1 r\sqrt{1-r^2} dr$

(4) (a) State the Fundamental Theorem of Calculus (both parts).

(b) Calculate the derivative of the function $h(x) = \int_0^{x^2} \cos \sqrt{\theta} d\theta$.

(c) Find the critical points of the function h given at point (b).

(5) (a) Find the average value of the function $f(x) = x^2 - 1$ on the interval $[0, \sqrt{3}]$.

(b) The Mean Value Theorem states that there is a value $x = c$ such that $f(c)$ equals to the average value of f . Find this value c .

Extra-credit: (20 points)

A boat is 4 miles from the nearest point on a straight shore line which is 6 miles from a shoreside restaurant. A woman plans to row to a point on shore, and then walk to the restaurant. If she can walk at 30 mph, at what speed must she be able to row so that the quickest way to get to the restaurant is to row directly? You may use the following guidelines:

- (a) Call v the unknown rowing speed.
- (b) Suppose the woman decides to row to a point on the shore x miles away from the restaurant. What is the distance d_1 she will be rowing? (Hint: Use Pithagora's Theorem.) What is the distance d_2 she will be walking?
- (c) What is the time T_1 spent rowing with the speed v ? What is the time T_2 spent walking with 30 mph? What is the total time $T = T_1 + T_2$? (Hint: your answer should depend on v and x .)
- (d) The total time T depends on the choice of x . Find when T is minimal by differentiating with respect to x and calculating the critical point. (Hint: When differentiating, think of v as a constant, that does not depend on x . Your answer will still depend on the speed v .)
- (e) Find the value of v by setting the critical point found at (d) to be equal to 6 mi.

Good luck!