

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your (1) name and (2) instructor's name on the front of your bluebook. Work all **5 problems**. Start each problem on a **new page**. Show your work clearly and **box** your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (25 points)

- a) Write an equation of a function that is continuous for all values of x except at $x = 2$ where it has a removable discontinuity and at $x = 5$ where it has a vertical asymptote.
- b) Does the curve $y = x^3$ ever have a negative slope? Use **calculus** to justify your answer.
- c) What angle θ (in radians) makes the following equations true? (*Show work with the unit circle and special triangles*)

$$\sin \theta = -\frac{1}{2} \quad \cos \theta = -\frac{\sqrt{3}}{2}$$

- d) Draw a counterexample to prove the following statement is not always true:

If $f(x)$ is an even function and we know $\lim_{x \rightarrow a^+} f(x) = b$, then we also know $\lim_{x \rightarrow a^-} f(x) = b$.

2. (25 points) Find the following limits, if they exist. If the limit does not exist or is infinity, clearly state this. Show all your work and justify your answers.

a) $\lim_{x \rightarrow 5} \frac{x^3 - 5x^2}{x^2 - 25}$

b) $\lim_{x \rightarrow 3} \frac{x^{10}}{(x-3)^2}$

c) $\lim_{x \rightarrow 0^+} 10 \sin\left(\frac{10}{x}\right)$

d) $\lim_{x \rightarrow 7} f(x)$ where $f(x) = \begin{cases} |7-x| & , x < 7 \\ x^2 - 5x - 14 & , x \geq 7 \end{cases}$

3. (25 points)

- a) State the definition of the derivative $f'(x)$.

b) Use this definition to calculate $f'(x)$ when $f(x) = \frac{1}{\sqrt{x}}$.

- c) Find an equation of the tangent line to $f(x)$ at $x = 4$.

4. (25 points) Find values of a and b such that

$$f(x) = \begin{cases} 1 - x^2 & , x < 0 \\ ax + b & , x \geq 0 \end{cases}$$

is differentiable at $x = 0$. **GRAPH** the function using the values of a and b you found. Justify why the function is differentiable at $x = 0$ by using the concepts of continuity and derivatives.

THERE IS ONE MORE ON THE BACK

5. (25 points) Consider the functions

$$f(x) = \cos(x) + 2$$

$$g(x) = \sqrt{x}$$

- a) State the domain and range for $f(x)$
- b) State the domain and range for $g(x)$.
- c) Is $f(x)$ even, odd, or neither? Justify
- d) Is $g(x)$ even, odd, or neither? Justify
- e) Does $f(x) = g(x)$ have at least one solution? Justify