① a) $y = \dfrac{x-2}{(x-2)(x-5)} = \dfrac{x-2}{x^2-7x+10}$ is one example

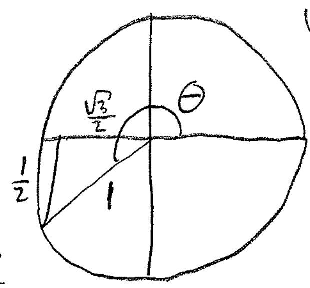
b) $y = x^3$

$y' = 3x^2 \rightarrow$ Anything squared is positive.
A negative squared is (+).

$\therefore 3x^2 > 0$ so the slope is always positive

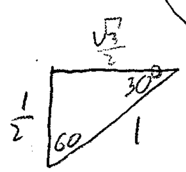
$y = x^3$ never has a negative slope

c) unit circle has $r = 1$



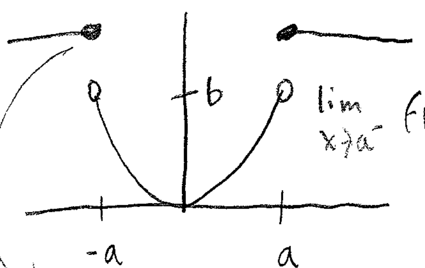
$\cos\theta = -\frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{1}{2}$



$180^\circ + 30^\circ = 210^\circ = \boxed{\frac{7\pi}{6}}$

d)



$\lim\limits_{x \to a^-} f(x) = b$

Here $\lim\limits_{x \to -a^-} f(x) \neq b$ and

$f(x)$ is still even

② a) $\lim\limits_{x \to 5} \dfrac{x^3-5x^2}{x^2-25} = \lim\limits_{x \to 5} \dfrac{x^2(x-5)}{(x-5)(x+5)} = \lim\limits_{x \to 5} \dfrac{x^2}{x+5} = \dfrac{25}{10} = \boxed{\dfrac{5}{2}}$

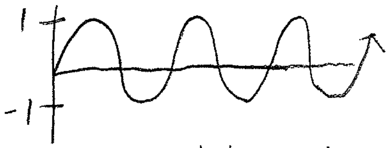
b) $\lim\limits_{x \to 3} \dfrac{x^{10}}{(x-3)^2}$

3^{10} is a finite number, and positive.
$(x-3)^2$ approaches $+0$ as x approaches 3 from the left and right
Any (+) number divided by a really small (+) number "blows-up."

$\therefore \lim\limits_{x \to 3} \dfrac{x^{10}}{(x-3)^2} = \infty$.

2c) $\lim_{x\to0^+} 10\sin\left(\frac{10}{x}\right) = 10\lim_{x\to0^+}\sin\left(\frac{10}{x}\right) = DNE$

Reason: $\lim_{x\to0^+}\frac{10}{x} = \infty$



sin(x) oscillates between -1 and 1. As x approaches ∞, sin(x) still oscillates in this range. $\therefore \lim_{x\to\infty}\sin(x)$ DNE, and likewise

$\lim_{x\to0^+} 10\sin\left(\frac{10}{x}\right)$ DNE.

2d) $\lim_{x\to7} f(x) = 0$

Reason:

$$\lim_{x\to7^-} f(x) = \lim_{x\to7^-}|7-x| = 0$$

$$\lim_{x\to7^+} f(x) = \lim_{x\to7^+} x^2-5x-14 = 49-35-14 = 0$$

Since the limit from the left equals the limit from the right, the overall limit also exists.

$$\lim_{x\to7} f(x) = 0$$

3) $f'(x) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$, provided the limit exists

$f(x) = \frac{1}{\sqrt{x}}$

$$f'(x) = \lim_{h\to0}\frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to0}\frac{\frac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}} - \frac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$ Common Denominator

$$= \lim_{h\to0}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right)$$

Multiply by Complex Conjugate

$$= \lim_{h\to0}\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$ cancel the h's, removing the divide by zero

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

Apply limit

$$f'(x) = \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})}$$

$$f'(x) = \frac{-1}{x(2\sqrt{x})} = \boxed{\frac{-1}{2x^{3/2}}}$$

c) at $x=4$

$$f(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$f'(4) = \frac{-1}{2(4)^{3/2}} = \frac{-1}{2(8)} = -\frac{1}{16}$$

$$\boxed{y - \frac{1}{2} = -\frac{1}{16}(x-4)}$$

4) For a function to be differentiable at a point, the function needs to be

- continuous at that point
- the left hand derivative has to equal the right hand derivative at that point, provided the limits exist

Continuous

$$1-x^2 = ax+b \text{ at } x=0$$

$$1-0^2 = a(0)+b$$

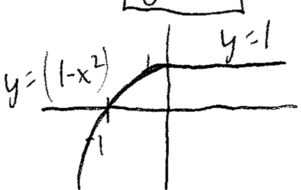
$$\boxed{1=b}$$

Derivatives

$$d/dx\,(1-x^2) = d/dx\,(ax+b) \text{ at } x=0$$

$$-2x = a \text{ at } x=0$$

$$\boxed{0=a}$$



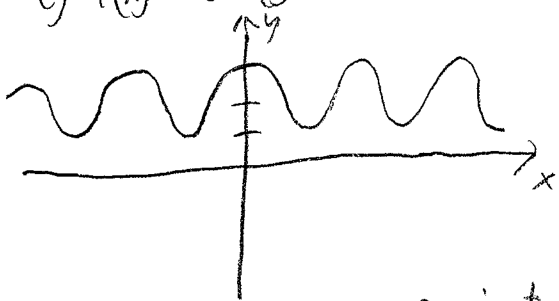
(5) a) Domain: $\mathbb{R}$ = all real numbers = $(-\infty, \infty)$

Range: $[1,3]$ - Note range of $\cos(x) = [-1,1]$

b) Domain: $x \geq 0 = [0, \infty)$

Range: $y \geq 0 : [0, \infty)$

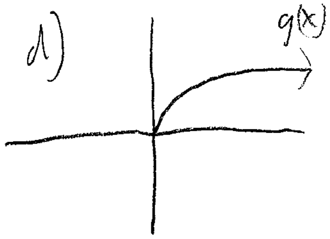
c) $f(x) = \cos(x) + 2$ is $\boxed{\text{Even}}$



$\cos(x)$ is even. The $+2$ just shifts $\cos(x)$ up 2, which doesn't change the symmetry over the y-axis. Also $f(x) = f(-x)$ for even so

$\cos(-x) + 2 = \cos(x) + 2$

d) $g(x) = \sqrt{x}$ is $\boxed{\text{neither}}$



$g(-x) = \sqrt{-x}$ is not a real number,

Therefore $g(x) \neq g(-x)$ for even and

$g(x) \neq -g(-x)$ for odd since $g(-x)$ is undefined in the real plane.

e) $f(x) = g(x)$

$\cos(x) + 2 = \sqrt{x}$

$f(x)$ is continuous for all values of x

$g(x)$ is continuous for $x \geq 0$

$\cos(x) + 2 - \sqrt{x} = 0$

At $x = 0$

$\cos(0) + 2 - \sqrt{0} = 1 + 2 = 3 > 0$

At $x = 36\pi$

$\cos(36\pi) + 2 - \sqrt{36\pi} = 1 + 2 - 6\sqrt{\pi}$

$= 3 - 6\sqrt{\pi} < 0$

By IVT, there is at least one solution to $f(x) = g(x)$ between $x = 0$ and $x = 36\pi$