

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your (1) name and (2) instructor's name on the front of your bluebook. Work all 5 problems. Start each problem on a **new page**. Show your work clearly and **box** your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (30 points) Using the appropriate rules of differentiation, find the derivative of the following functions. Simplify where appropriate.

a) $g(t) = 4t^2 \sqrt{5t-1}$

d) $k(n) = \cos^2 n + \sin^2 n$

b) $f(x) = \frac{\sin x}{\sin x + \cos x}$

e) What is $\frac{dy}{dx}$ if $y = 6u^2$ and $u = \frac{\sqrt{x}}{2}$

c) $r(\theta) = \tan^3\left(\frac{1}{\theta^3}\right)$

2. (18 points) Find the following limits, if they exist. If the limit does not exist or is infinity, clearly state this. Show all your work and justify your answers.

a) $\lim_{x \rightarrow -\infty} \frac{8x^2 - 2x + \cos(5x)}{3x^2}$

b) $\lim_{x \rightarrow 0} \frac{\sin^2(7x) - \sin(7x)}{x}$

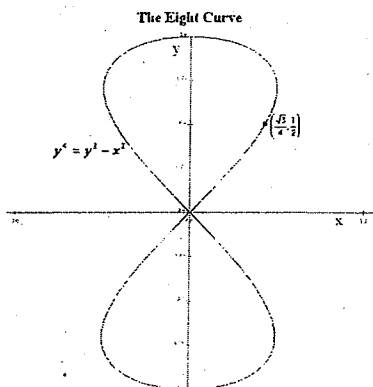
c) $\lim_{x \rightarrow \infty} \frac{4\sqrt{x} - \sqrt[3]{x}}{8-x}$

3. (22 points)

a) Explain why the fourth derivative of a quintic (fifth order) polynomial is a line with a non-zero slope.

b) I went camping in the Great Sand Dunes last weekend. Let's say the dunes are 240 miles away from Boulder and it took me four hours to drive there. Did I ever go exactly 60 mph on the trip? Justify, naming a theorem and explaining why it is appropriate in this scenario.

c) Write an equation for the tangent line to the eight curve $y^4 = y^2 - x^2$ (see below) at the point $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$



4. (25 points) Elmer Fudd is running at 26 ft/sec south towards a street intersection while Bugs Bunny is running east, away from the intersection, at 13 ft/sec. When Fudd is 5 feet from the intersection, Bugs is 12 feet from the intersection. How rapidly is the distance between Fudd and Bugs changing?

5. (30 points)

Consider the function $f(x) = \frac{2x^2 + 4}{x}$

- a) Is $f(x)$ even, odd, or neither? Show work
- b) If $f(x)$ has any asymptotes, classify them and give the equations of the asymptotes.
- c) Identify the interval(s) on which the function is increasing and decreasing.
- d) Identify the interval(s) on which the function is concave up and concave down.
- e) Find all local maxima and minima.
- f) Make a sketch of the function on which you include and **LABEL** asymptotes, extreme values and inflection points.