1) a) $g'(t) = 8t\sqrt{5t-1} + \frac{1}{2\sqrt{5t-1}}(5)(4t^2)$

$$\boxed{g'(t) = 8t\sqrt{5t-1} + \frac{10t^2}{\sqrt{5t-1}}}$$

b) $f'(x) = \frac{\cos(x)(\sin x + \cos x) - (\cos(x) - \sin(x))\sin(x)}{(\sin x + \cos x)^2}$

$f'(x) = \frac{\cos x \sin x + \cos^2 x - \cos x \sin x + \sin^2 x}{\sin^2 x + 2\sin x \cos x + \cos^2 x}$

$f'(x) = \frac{\sin^2 x + \cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$

$$\boxed{f'(x) = \frac{1}{2\sin x \cos x}}$$

c) $r'(\theta) = 3\tan^2\left(\frac{1}{\theta^3}\right) \cdot \sec^2\left(\frac{1}{\theta^3}\right)(-3\theta^{-4})$

$$\boxed{r'(\theta) = \frac{-9\tan^2\left(\frac{1}{\theta^3}\right)\sec^2\left(\frac{1}{\theta^3}\right)}{\theta^4}}$$

d) $\boxed{k'(n) = 0}$ because

$k(n) = \cos^2 n + \sin^2 n$

$k(n) = 1$

$k'(n) = 0$

or

$k'(n) = 2\cos(n)(-\sin(n)) + 2\sin(n)\cos(n)$

$= -2\cos(n)\sin(n) + 2\cos(n)\sin(n)$

$\boxed{k'(n) = 0}$

e) $y = 6u^2$ and $u = \frac{\sqrt{x}}{2}$

$y = 6\left(\frac{\sqrt{x}}{2}\right)^2$

$y = \frac{6x}{4}$

$\boxed{dy/dx = 6/4 = 3/2}$

or Chain Rule

$dy/dx = dy/du \cdot du/dx$

$dy/dx = (12u)\left(\frac{1}{4\sqrt{x}}\right)$

$dy/dx = \left(\frac{12\sqrt{x}}{2}\right)\left(\frac{1}{4\sqrt{x}}\right)$

$\boxed{dy/dx = 3/2}$

(2)

a) $\lim\limits_{x\to-\infty} \dfrac{8x^2-2x+\cos(5x)}{3x^2}$

$\lim\limits_{x\to-\infty} \dfrac{8}{3} - \lim\limits_{x\to-\infty} \dfrac{2}{3x} + \lim\limits_{x\to-\infty} \dfrac{\cos 5x}{3x^2}$

$\dfrac{8}{3} - 0 + 0 = \boxed{8/3}$

Note: $\lim\limits_{x\to-\infty} \dfrac{-1}{3x^2} < \lim\limits_{x\to-\infty} \dfrac{\cos 5x}{3x^2} < \lim\limits_{x\to-\infty} \dfrac{1}{3x^2}$

$0 < \lim\limits_{x\to-\infty} \dfrac{\cos 5x}{3x^2} < 0$

$\therefore \lim\limits_{x\to-\infty} \dfrac{\cos 5x}{3x^2} = 0$ by Sandwich thm.

b) $\lim\limits_{x\to0} \dfrac{\sin^2(7x)-\sin(7x)}{x}$

$\lim\limits_{x\to0} \dfrac{\sin(7x)\left(\sin(7x)-1\right)}{x}$

$\lim\limits_{x\to0} \left(\dfrac{7\sin 7x}{7x}\right)\left(\sin(7x)-1\right)$

$\lim\limits_{x\to0} 7(\sin 7x - 1) = 7(0-1) = \boxed{-7}$

c) $\lim\limits_{x\to\infty} \dfrac{4\sqrt{x} - \sqrt[3]{x}}{8-x}$

$\lim\limits_{x\to\infty} \dfrac{\frac{4\sqrt{x}}{x} - \frac{\sqrt[3]{x}}{x}}{\frac{8}{x} - \frac{x}{x}}$

$\lim\limits_{x\to\infty} \dfrac{\frac{4}{x^{1/2}} - 1/x^{2/3}}{\frac{8}{x} - 1}$

$\lim\limits_{x\to\infty} \dfrac{0-0}{0-1} = \boxed{0}$

③a) $y = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$
where $a_5 \neq 0$ because it's a fifth order

$y' = 5a_5x^4 + 4a_4x^3 + 3a_3x^2 + 2a_2x + a_1$

$y'' = 20a_5x^3 + 12a_4x^2 + 6a_3x + 2a_2$

$y''' = 60a_5x^2 + 24a_4x + 6a_3$

$y^{(4)} = 120a_5x + 24a_4$ which is in the form $y = mx + b$ where $m = 120a_5$ and $b = 24a_4$

This is a line with a non-zero slope

b) Driving a car is a smooth, continuous excursion. Thus in plotting the distance traveled over time, I know it's continuous and differentiable on my interval. Therefore the Mean Value theorem holds, which states

$\frac{f(b) - f(a)}{b-a} = f'(c)$ for at least one point $x = c$ in my interval.

the average speed will be an instantaneous speed at some point during my trip. Here my average speed is $\frac{240}{4} = 60$mph. Thus somewhere on the trip I was going exactly 60 mph.

c). $y^4 = y^2 - x^2$
Implicit Differentiation

$4y^3y' = 2yy' - 2x$

$4y^3y' - 2yy' = -2x$

$y'(4y^3 - 2y) = -2x$

$y' = \frac{-2x}{4y^3 - 2y}$

$\boxed{y' = \frac{-x}{2y^3 - y}}$

Need tangent line through $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$

slope $y' = \frac{-\left(\frac{\sqrt{3}}{4}\right)}{2(1/2)^3 - 1/2}$

$y' = \frac{-\frac{\sqrt{3}}{4}}{2(1/8) - 1/2}$

$y' = \frac{-\frac{\sqrt{3}}{4}}{1/4 - 1/2}$

$y' = \frac{-\frac{\sqrt{3}}{4}}{-\frac{1}{4}}$

$\boxed{y' = \sqrt{3}}$

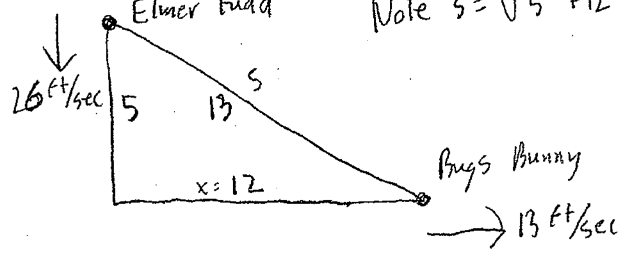
Tangent Line $f(x)$
using Pt-slope eqn.

$f(x) = \frac{1}{2} + \sqrt{3}\left(x - \frac{\sqrt{3}}{4}\right)$

$f(x) = \frac{1}{2} + \sqrt{3}x - \frac{3}{4}$

$\boxed{f(x) = \sqrt{3}x - 1/4}$

(4)



Note $s = \sqrt{5^2+12^2} = \sqrt{169} = 13$

Want ds/dt

$$x^2 + y^2 = s^2$$
$$2x\, dx/dt + 2y\, dy/dt = 2s\, ds/dt$$
$$x\, dx/dt + y\, dy/dt = s\, ds/dt$$
$$12(13) + (5)(-26) = 13\, ds/dt$$
$$156 - 130 = 13\, ds/dt$$
$$26 = 13\, ds/dt$$
$$\boxed{ds/dt = 2 \text{ ft/sec}}$$

(5) $f(x) = \dfrac{2x^2+4}{x} = 2x + \dfrac{4}{x}$

a) $f(-x) = \dfrac{2(-x)^2+4}{-x} = \dfrac{2x^2+4}{-x} = -\left(\dfrac{2x^2+4}{x}\right) = -f(x)$

$f(x)$ is therefore $\boxed{\text{odd}}$

b) Vertical Asymptote at $x = 0$
Oblique Asymptote at $y = 2x$
There is no horizontal asymptote.

c) $f(x) = 2x + \dfrac{4}{x}$

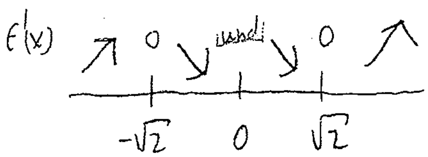
$f'(x) = 2 - \dfrac{4}{x^2} = 0$

$2 = \dfrac{4}{x^2}$

$x^2 = 2$

$x = \pm\sqrt{2}$

Function is increasing in $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$
Function is decreasing in $(-\sqrt{2}, 0)$ and $(0, \sqrt{2})$



(5 cont)

d) $y'' = 8/x^3 = 0$ never so no inflection point

$$\begin{array}{ccc} (-) & & (+) \\ \hline & 0 & \\ & \text{V.A} & \end{array}$$

Function is concave down in $(-\infty, 0)$
function is concave up in $(0, \infty)$

e) $f'(x) = 0$ at $x = \pm\sqrt{2}$

$f(\sqrt{2}) = \dfrac{2(\sqrt{2})^2+4}{\sqrt{2}} = \dfrac{8}{\sqrt{2}} = 4\sqrt{2}$ is a local min b/c function is concave up

$f(-\sqrt{2}) = -4\sqrt{2}$ is a local max b/c function is concave down

Local min at $(\sqrt{2}, 4\sqrt{2})$
Local max at $(-\sqrt{2}, -4\sqrt{2})$

f)