

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your (1) name and (2) instructor's name on the front of your bluebook. Work all **6 problems**. Start each problem on a **new page**. Show your work clearly and **box** your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (45 points) Find the following limits, if they exist. If the limit does not exist or is infinity, clearly state this. Show all your work and justify your answers.

a) $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 + 1} - \sqrt{2x}}{(x-1)}$

b) $\lim_{x \rightarrow 0} \frac{x - \sin x}{1 - \cos x}$

c) $\lim_{x \rightarrow \infty} \frac{x^6}{6e^x}$

2. (45 points) Take the following derivatives

a) $y = (1 + 2x)e^{-2x}$

b) $y = \tan^{-1}(\sqrt{x})$

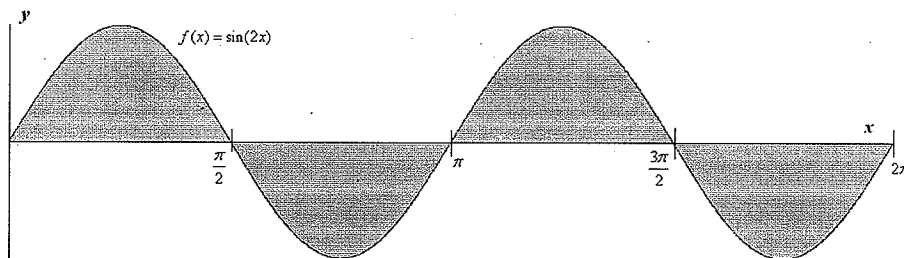
c) $y = x^x$

3. (45 points) Evaluate the following integrals

a) $\int e^{(x^2 - 4x + 7)}(x - 2) dx$

b) $\int_2^6 \frac{4x + 4}{x^2 + 2x - 3} dx$ (Hint: Complete answer will be reduced to one number)

c) Find the shaded area of



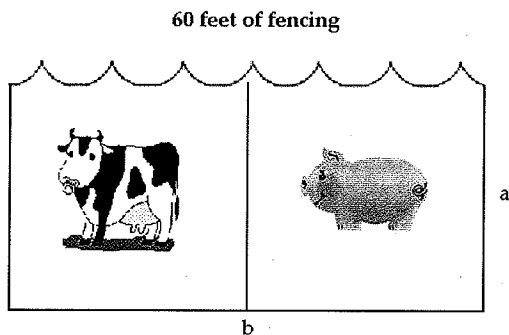
THERE ARE THREE MORE ON THE BACK

4. (30 points) Find values of a and b such that

$$f(x) = \begin{cases} x^2 + ax + b, & x < 0 \\ \sin x, & x \geq 0 \end{cases}$$

is differentiable at $x = 0$. **GRAPH** the function using the values of a and b you found. Justify why the function is differentiable at $x = 0$ by using the concepts of continuity and derivatives.

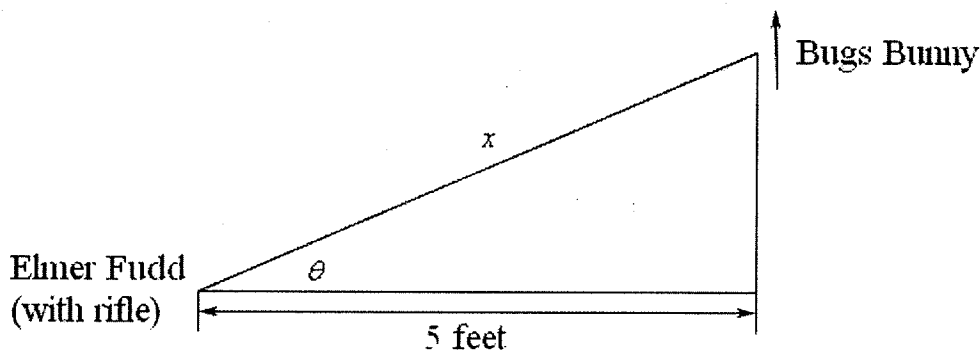
5. (30 points) **Optimization:**



Farmer Brown needs to build a rectangular pen for his cow and pig. He has 60 feet of fencing available to use along with the river cutting through his farm. The two animals have to be separated from each other. They are both afraid of the water, so they won't escape across the river. What is the maximum TOTAL area (cow pen plus pig pen) that Farmer Brown can fence in?

6. (30 points) **Related Rates:** Bugs Bunny and Elmer Fudd are 5 feet apart. Bugs is running north such that Fudd has to rotate his gun through the angle θ shown below at a rate of $\frac{20}{\sqrt{3}}$ rad/min.

Fudd has Bugs in his rifle's scope. Now he just needs to determine at what rate the distance between the two is changing when Bugs is $x = 10$ feet away. Find $\frac{dx}{dt}$.



Hint: Relate θ and x in an equation and take the derivative with respect to time. You do know θ from the information given!