Calculus 1

Quiz 1

Show work!

- Find the following limit. (Hint: $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$) $\lim_{\theta \to 0} \frac{\tan 12\theta}{\theta}$

$$\lim_{\theta \to 0} \frac{\tan 12\theta}{\theta} = \lim_{\theta \to 0} \frac{\sin(12\theta)}{\cos(12\theta)\,\theta} \cdot 12 = \lim_{\theta \to 0} \frac{12}{\cos(12\theta)} = 12$$

($\sin(12\theta) \to 1$, $\cos(12\theta) \to 1$)

- Find $\frac{dy}{dx}$ (Hint: Use product rule and chain rule):

$$y = \cos\left(x^2\right) x$$

$$\frac{dy}{dx} = -\sin(x^2)(2x)(x) + \cos(x^2)$$

$$\frac{dy}{dx} = -2x^2 \sin(x^2) + \cos(x^2)$$

- Find $\frac{dy}{dx}$

$$y = x\tan(x) + x^{-2} + 3$$

$$\frac{dy}{dx} = \tan x + x\sec^2(x) - 2x^{-3}$$

$$\frac{dr}{d\theta}(\theta + \cos(r\theta)\theta) = -r - r\cos(r\theta)$$

$$\frac{dr}{d\theta} = \frac{-r(1+\cos(r\theta))}{\theta(1+\cos(r\theta))} = \boxed{\frac{-r}{\theta}}$$

- Find $\frac{dr}{d\theta}$ (Hint: Use implicit differentiation):

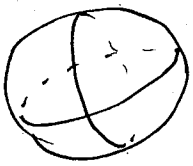
$$r\theta + \sin(r\theta) = 4$$

$$\frac{dr}{d\theta} \quad \frac{d}{d\theta}(r\theta) + \frac{d}{d\theta}(\sin(r\theta)) = \frac{d}{d\theta}4$$

$$\frac{dr}{d\theta}\theta + r\frac{d\theta}{d\theta} + \cos(r\theta)\left(\frac{dr}{d\theta}\theta + r\right) = 0$$

$$\frac{dr}{d\theta}\theta + r + \cos(r\theta)\frac{dr}{d\theta}\theta + r\cos(r\theta) = 0$$

- James Bond is inflating a hot air balloon (perfectly sphere) at a rate of 30 m^3/ second to escape Dr. Evil. How fast is the radius changing when the radius is 10 m? James Bond wants you to solve for $\frac{dr}{dt}$, but does not need you to simplify (Hint: $v = \frac{4}{3}\pi r^3$ and a picture might help!)



$$\frac{dV}{dt} = 30\,m^3/sec$$

$r = 10\,m$ ← Find $\frac{dV}{dt}$ when $r=10m$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$30\,m^3/sec = 4\pi(10)^2\frac{dr}{dt}$$

$400\pi\,m^2$

$$\frac{30\,m^3/sec}{400\pi\,m^2} = \frac{dr}{dt} = \boxed{\frac{3}{40\pi}\,m/sec}$$