

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) student number, (3) instructor's name, and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. This exam has 6 problems, on both sides of this sheet. Work all 6 problems. Start each problem on a new page. Show your work. Box in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points)

a) Find the derivative of $v(t) = (1-t)(1+t^2)^{-1}$. Then evaluate the derivative at $t = 0$.

b) Find the first and second derivatives of $w(z) = \left(\frac{1+3z}{3z}\right)(3-z)$.

2. (25 points)

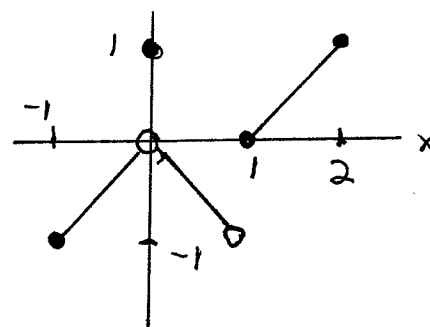
a) State the definition of the derivative of a function, $f(x)$, at a point x in its domain.

b) Let $f(x) = 1 + \sqrt{x-1}$. Sketch the graph of $y = f(x)$ in its domain.

c) Use the definition of a derivative to find $\frac{df}{dx}$, for $f(x)$ defined in (b), above.

d) Find the equation of the line tangent to $y = f(x)$ at $x = 2$, if there is one.

3. (15 points) Which of the statements about the function graphed here are true and which are false? After answering that, replace ONE of the false statements with a true statement about $y = f(x)$.



a) $\lim_{x \rightarrow 0} f(x)$ exists b) $\lim_{x \rightarrow 0} f(x) = 0$ c) $\lim_{x \rightarrow 1^-} f(x) = 0$

d) $\lim_{x \rightarrow x_0} f(x)$ exists at every point in $(-1, 1)$

e) $f(x)$ is continuous at every point in $(-1, 1)$

4. (15 points)

a) What is required for a function $f(x)$ to be continuous at $x = x_0$? Be specific.

b) Suppose $f(x)$ satisfies $|f(x)| \leq x^2$, $-1 < x < 1$. Is $f(x)$ necessarily continuous at $x = 0$? If so, write YES, explain why, and find $f(0)$. If not, write NO, and give a counterexample (*i.e.*, a function that satisfies the inequality but is not continuous at $x = 0$).

5. (15 points)

Suppose that u and v are functions of x that are differentiable at $x = 0$, and that

$$u(0) = 5, \quad u'(0) = -3, \quad v(0) = -1, \quad v'(0) = 2.$$

Find the values of the following derivatives at $x = 0$.

a) $\frac{d}{dx}(uv)$

b) $\frac{d}{dx}\left(\frac{u}{v}\right)$

c) $\frac{d}{dx}(7v - 2u)$.

6. (15 points)

At time t , the position of a body moving along the s -axis is $s = t^3 - 6t^2 + 9t$.

- Find the body's acceleration each time the velocity is zero.
- When is the body moving forward? When is it moving backward?
- When is the body's velocity increasing?

