APPM 1350 – Exam 1 – Answers

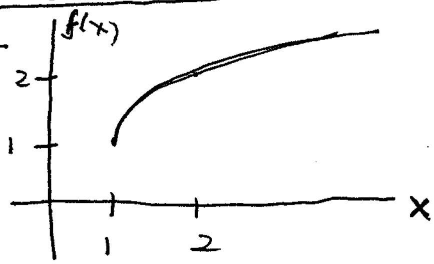
1. a) $V(t) = \frac{1-t}{1+t^2}$. $\frac{dV}{dt} = \frac{(1+t^2)(-1)-(1-t)(2t)}{(1+t^2)^2} = \boxed{\frac{t^2-2t-1}{(1+t^2)^2}}$ At $t=0$, $\boxed{\frac{dV}{dt}(0) = -1}$

b) $w(z) = \left(\frac{1+3z}{3z}\right)(3-z) = \left(\frac{1}{3z}+1\right)(3-z) = \frac{1}{z} + 3 - \frac{1}{3} - z = z^{-1} + \frac{8}{3} - z$

$\frac{dw}{dz} = -z^{-2} + 0 - 1 = \boxed{-\frac{1}{z^2} - 1 = \frac{dw}{dz}}$ $\boxed{\frac{d^2w}{dz^2} = 2z^{-3} = \frac{2}{z^3}}$ $\boxed{\text{Homework: §2.2, \#21, 35}}$

2. a) $\boxed{\frac{df}{dx}(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$ if the limit exists

b) $f(x) = 1+\sqrt{x-1}$



c) $\frac{df}{dx} = \lim_{h\to 0} \frac{1+\sqrt{x+h-1} - (1+\sqrt{x-1})}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{x+h-1}-\sqrt{x-1})(\sqrt{x+h-1}+\sqrt{x-1})}{h(\sqrt{x+h-1}+\sqrt{x-1})}$

$= \lim_{h\to 0} \frac{(x+h-1)-(x-1)}{h(\sqrt{x+h-1}+\sqrt{x-1})}$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-1}+\sqrt{x-1})}$

$\boxed{\frac{df}{dx} = \frac{1}{2\sqrt{x-1}}}$ $\boxed{\text{Homework: Part(b) was §P4, 2)}}$

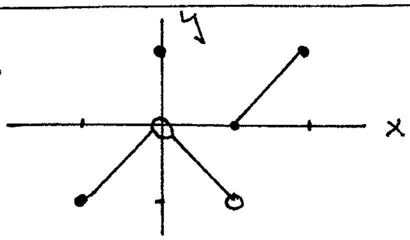
d) At $x=2$, $f(2) = 2$, $\frac{df}{dx} = \frac{1}{2}$

tangent line:

$y - y_0 = (m)(x-x_0)$

$\boxed{y-2 = \frac{1}{2}(x-2)}$ or $\boxed{y = \frac{1}{2}x+1}$

3.



a) $\lim_{x\to 0} f(x)$ exists **T**

b) $\lim_{x\to 0} f(x) = 0$ **T**

c) $\lim_{x\to 1^-} f(x) = 0$ **F**

d) $\lim_{x\to x_0} f(x) =$ exists at every point in $(-1,1)$ **F**

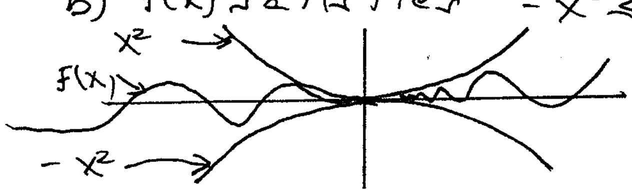
e) $f(x)$ is continuous at every point in $(-1,1)$ **F**

Many replacement statements are possible!
examples: $\lim_{x\to 1^-} f(x) = -1$, or $f(x)$ is continuous in $(0,1)$ or $f(0) = 1$ or ...

$\boxed{\text{Homework: (a), (b), (d) come from §1.1, \#3}}$

4) (a) $f(x)$ is continuous at $x = x_0$ if:
(i) $f(x_0)$ is defined; (ii) $\lim_{x\to x_0} f(x)$ exists; (iii) $\lim_{x\to x_0} f(x) = f(x_0)$.

b) $f(x)$ satisfies $-x^2 \le f(x) \le x^2$, $-1 < x < 1$. $f(x)$ __is__ continuous at $x=0$, so $\boxed{\text{YES}}$. Because (i) $f(0) = 0$, & (ii) $\lim_{x\to 0} f(x) = 0$ by sandwich Th'm



5. $u(0)=5$, $u'(0)=-3$, $v(0)=-1$, $v'(0)=2$

a) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$. At $x=0$, $\frac{d}{dx}(uv) = (5)(2)+(-1)(-3)$

$= 10+3 = \boxed{13}$

b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{(v)^2}$. At $x=0$, $\frac{d}{dx}\left(\frac{u}{v}\right) = \dfrac{(-1)(-3)-(5)(2)}{(-1)^2}$

$= 3-10 = \boxed{-7}$

c) $\frac{d}{dx}(7v-2u) = 7\frac{dv}{dx} - 2\frac{du}{dx}$. At $x=0$, $\frac{d}{dx}(7v-2u) = (7)(2)-2(-3)$

$= 14+6 = \boxed{20}$

Homework: §2.2, #39

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6. Position: $s(t) = t^3 - 6t^2 + 9t$

$\Rightarrow$ velocity: $v = \frac{ds}{dt} = 3t^2 - 12t + 9 = 3(t^2-4t+3)$

$\Rightarrow$ acceleration: $a = \frac{dv}{dt} = 6t - 12$

a) velocity $= 0 \Rightarrow 3(t^2-4t+3) = 3(t-1)(t-3) = 0$

$v = 0$ at $t=1$ & at $t=3$

At $t=1$, $a = 6-12 = \boxed{-6 = a}$ At $t=3$, $a = 6(3)-12 = \boxed{6 = a}$

b) Body moves forward when $v(t) > 0$

Either $\boxed{t>3, \text{ or } t<1}$

Body moves backwards when $v(t) < 0 \Rightarrow \boxed{1<t<3}$

c) Body's velocity is increasing when $a(t) > 0$

$6t - 12 > 0 \Rightarrow \boxed{t>2}$

Average on this exam: Depending on your Lecture section, in the range of 73-75.