SOLUTION SET EXAM 3
APPM 1350

Problem 1

$$\int_{-1}^{5} 1-2f(x)\,dx = -6 \qquad \& \qquad \int_2^5 f(x)+3\,dx = 13$$

$$\int_{-1}^{5} dx - 2\int_{-1}^{5} f(x)\,dx = -6 \qquad \int_2^5 f(x)\,dx + \int_2^5 3\,dx = 13$$

$$5-(-1) - 2\int_{-1}^{5} f(x)\,dx = -6 \qquad \int_2^5 f(x)\,dx + 3(5-2) = 13$$

$$\int_{-1}^{5} f(x)\,dx = 6 \qquad \int_2^5 f(x)\,dx = 4$$

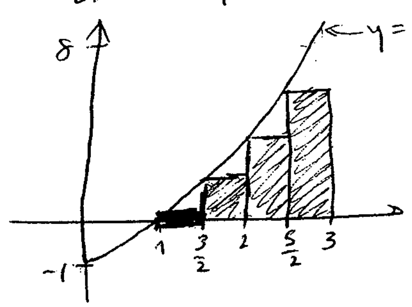
$$\Rightarrow \boxed{\int_{-1}^{2} f(x)\,dx = \int_{-1}^{5} f(x)\,dx - \int_2^5 f(x)\,dx = 6-4 = 2}$$

Problem 2

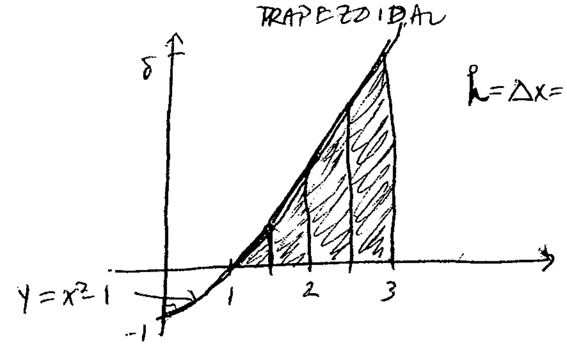
$n=4$ equally spaced partitions

Ⓐ

LEFT ENDPOINT



TRAPEZOIDAL



$$h = \Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

Ⓑ LEFT

$$L_4 = \Delta x\left[f(1)+f(\tfrac{3}{2})+f(2)+f(\tfrac{5}{2})\right] = \frac{1}{2}\left[0+\left((\tfrac{3}{2})^2-1\right)+3+\left((\tfrac{5}{2})^2-1\right)\right]$$

$$= \frac{1}{2}\left[\frac{5}{4}+\frac{12}{4}+\frac{21}{4}\right] = \boxed{\frac{19}{4}}$$

Trapezoidal rule

$$T_4 = \frac{\Delta x}{2}\left[y_0+2y_1+2y_2+2y_3+y_4\right], \qquad y_0=f(1),\ y_1=f(3/2),\ y_2=f(2),\ y_3=f(5/2),\ y_3=f(3)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\left[0+2(5/4)+6+2(21/4)+8\right],$$

$$= \frac{1}{4}\left[10/4 + 24/4 + 42/4 + 32/4\right] = \boxed{27/4}$$

Ⓒ TRUE VALUE OF $\int_1^3 x^2-1\,dx = \frac{x^3}{3}-x\Big|_1^3 = \frac{27}{3}-3-\left[1/3-1\right] = \boxed{\frac{20}{3}}$

$$\text{Error} = \text{True Value} - T_4 = \frac{20}{3} - \frac{27}{4} = \boxed{\frac{1}{12}}$$

$$\text{Error Estimate} = \frac{b-a}{12}\cdot h^2 M, \qquad M = f''(x),\ \text{maximum over } [1,3]$$

$$f(x)=x^2-1,\ f'(x)=2x,\ f''(x)=2$$

$$\therefore M=2$$

$$= \frac{3-1}{12}\left(\frac{1}{2}\right)^2\cdot 2 = \boxed{\frac{1}{12}}$$

Problem 3

(a) $\int x^{-3}(x+1)\,dx$

$$= \int x^{-2}+x^{-3}\,dx$$

$$= \frac{x^{-2+1}}{-2+1} + \frac{x^{-3+1}}{-3+1} + C$$

$$= \boxed{-x^{-1} - \frac{1}{2}x^{-2} + C}$$

(b) $\int \frac{dx}{\sqrt{5x+8}}$ $\qquad u=5x+8,\ du=5dx$

$$= \int \frac{\frac{1}{5}du}{\sqrt{u}}$$

$$= \frac{1}{5}\int \frac{du}{\sqrt{u}}$$

$$= \frac{1}{5}\int u^{-1/2}\,du =$$

$$= \frac{1}{5}\,\frac{u^{-1/2+1}}{-1/2+1} + C$$

$$= \frac{2}{5}u^{1/2} + C$$

$$= \boxed{\frac{2}{5}\sqrt{5x+8} + C}$$

Problem 3 continued

(c) $\int_0^{\pi/4} \tan x\, dx = \int_0^{\pi/4} \frac{\sin x}{\cos x} dx = \int_0^{\pi/4} \frac{-du}{u} = -\int_1^{\sqrt{2}/2} \frac{du}{u}$

$u = \cos x \Rightarrow du = -\sin x\, dx$

at $x=0 \Rightarrow u = \cos 0 = 1$

at $x = \pi/4 \Rightarrow u = \cos \pi/4 = \sqrt{2}/2$

$= \int_{\sqrt{2}/2}^{1} \frac{1}{u} du = \ln|u| \Big|_{\sqrt{2}/2}^{1}$

$= \ln 1 - \ln \frac{\sqrt{2}}{2} = \boxed{-\ln \frac{\sqrt{2}}{2}}$

Problem 4 Fundamental Thm of Calculus

(a) Part I If f is continuous on $[a,b]$. Then $F(x) = \int_a^x f(t)dt$ has a derivative at every point of $[a,b]$ and

$$\frac{d}{dx} F(x) = \frac{d}{dx} \int_a^x f(t)dt = f(x), \quad a \le x \le b$$

Part II If f is continuous on $[a,b]$ and F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

(b) Linearization of $g(x)$ at $x=4$

$g(4) = 3 - \int_2^{\sqrt{4}} \sin(\pi t^2)dt = 3 - \int_2^2 \sin(\pi t^2)dt = 3$

$g'(x) = \sin(\pi(\sqrt{x})^2) \cdot \frac{1}{2} x^{-1/2} = \frac{1}{2}\sin(\pi x) \cdot x^{-1/2}$

$\Rightarrow g'(4) = \frac{1}{2}\sin(4\pi) \cdot (4)^{-1/2} = 0$

$L(x) = g(4) + g'(4)(x-4) = 3 + 0(x-4) = 3 \Rightarrow \boxed{L(x) = 3}$

Problem 5

(a) $y = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}} = \left[\frac{(x+1)^{10}}{(2x+1)^5}\right]^{1/2} \Rightarrow \ln y = \frac{1}{2}\ln\left(\frac{(x+1)^{10}}{(2x+1)^5}\right)$

$\ln y = \frac{1}{2}\left[\ln (x+1)^{10} - \ln(2x+1)^5\right] = \frac{1}{2}\left[10\ln(x+1) - 5\ln(2x+1)\right]$

$\frac{d}{dx}\ln y = \frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}\left[10\ln(x+1) - 5\ln(2x+1)\right]\right)$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\cdot\left[\frac{10}{x+1} - \frac{5\cdot 2}{2x+1}\right] = \frac{5}{x+1} - \frac{5}{2x+1}$

$dy/dx = y\left[\frac{5}{x+1} - \frac{5}{2x+1}\right] \Rightarrow \boxed{\frac{dy}{dx} = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}\left(\frac{5}{x+1} - \frac{5}{2x+1}\right)}$

(b) at $x=0$ $dy/dx = \sqrt{\frac{1^{10}}{1^5}}\left(\frac{5}{1} - \frac{5}{1}\right) = 0 \Rightarrow \boxed{dy/dx = 0 \text{ at } x=0}$

Problem 6

(a) $f(x) = x^3 - 1$ is one-to-one because it satisfies horizontal line test.

$g(x) = x^4 + 2$ is not one-to-one, $g(x)$ does not satisfy horizontal line test

Therefore $f(x)$ has an inverse. $y = (x+1)^{1/3} \Rightarrow \boxed{f^{-1}(x) = (x+1)^{1/3}}$

$y = x^3 - 1 \Rightarrow x = y^3 - 1 \Rightarrow y^3 = x+1 \Rightarrow$

(b) Graph of f and f^{-1}