**APPM 1350 FINAL EXAM Fall 2007**

**INSTRUCTIONS: This exam has 7 problems, each worth 25 points. Do any 6. Mark on the front of your exam which problem you are skipping. If no problem is identified on the front of the blue book, the first 6 problems will be graded.**

**Books, notes, flying monkeys and electronic devices are not permitted. Write on the front of your bluebook: (1) your name, (2) student number, (3) instructors name, (4) when your lecture meets; (5) which problem you are skipping. Also make a scoring table, with places for 7 problems, plus a total score. Show your work. Box in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.**

1. *Continuity problems*

   (a) If $f(x)$ is defined for all $x$, what is required for $f(x)$ to be continuous at $x = x_0$? Be specific.

   (b) Let
   $$g(x) = \begin{cases} x \cdot \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 1, & x = 0 \end{cases}$$
   Is $g(x)$ continuous at $x = 0$? Justify your answer.

   (c) Suppose $f(x) = \dfrac{1}{x-2}$ and $g(x) = x + 2$. Where is $f(x)$ continuous? Where is $g(x)$ continuous? Where is $(f \circ g)(x)$ continuous?

   (d) Give an example of two functions, $F(x)$ and $G(x)$, both continuous at $x = 0$, for which the composite $F \circ G$ is discontinuous at $x = 0$.

2. *Approximations of* $\ln(z)$

   (a) Find the linearization of $\ln(1 + x)$, valid near $x = 0$.

   (b) Based on your result in (a), find an approximate value for $\ln(1.1)$.

   (c) Write $\ln(1 + x)$ in terms of a definite integral whose limit(s) depend on $x$.

   (d) Use the trapezoidal rule (given below) with $N = 1$ to find a different approximate value for $\ln(1.1)$.

   (e) Use the error estimate for the trapezoidal rule to find an upper bound on the error of your approximation in (d).

3. *Logarithms and inverse trigonometric functions*

   (a) Find the derivative of $z(x) = \ln(\tan^{-1} x)$.

   (b) Find the derivative of $y(x) = \tan^{-1}(\ln x)$.

   (c) Which is bigger: $z(1)$ or $y(1)$? Justify your answer.

   (d) Find the equation of the line tangent to $y(x)$ from part (b), at $x = 1$.

   (e) Evaluate $\displaystyle\int_{-2}^{2} \frac{dt}{4 + 3t^2}$.

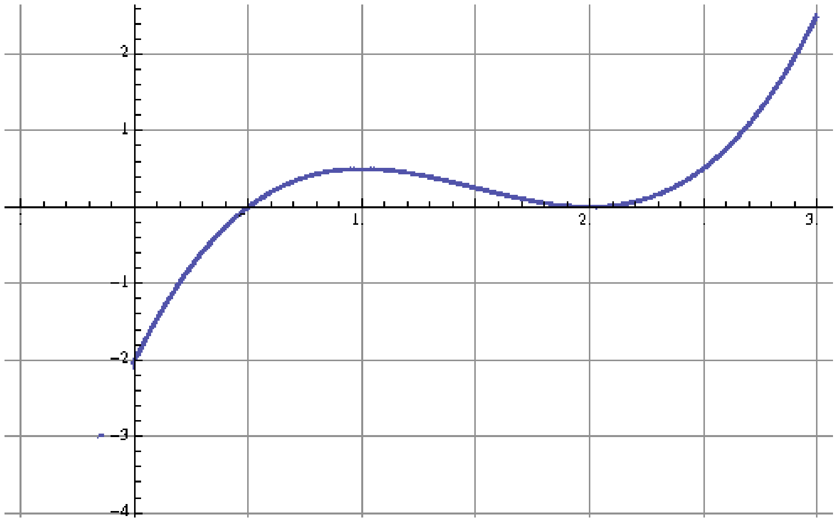


Figure 1: **The graph of $\frac{dg}{dx}$.**

4. *Graphs*

   The function $g(x)$ is defined on the interval $[0, 3]$. The graph of its derivative is shown in Figure 1.

   (a) Find the local minimum and local maximum points of $g(x)$, if any.

   (b) Find the region(s) where $g(x)$ is concave up. Find the region(s) where $g(x)$ is concave down. Be sure to identify which is which.

   (c) Find the inflection points of $g(x)$, if any.

   (d) Assume $g(0) = 0$. Sketch the graph of $g(x)$ for $0 \leq x \leq 3$.

5. *Optimization*

   A 4-m length of wire is available for making a circle and a square.

   (a) How should the wire be distributed between the two shapes to maximize the sum of the two areas?

   (b) What is the maximum enclosed area?

   (c) Justify that your answer is a maximum.

6. *Exponentials and logarithms*

   (a) Find numbers $x$ and $y$ such that

   $$e^x = 2 \quad \text{and} \quad 2^y = e.$$

   (b) What is the relation between $x$ and $y$?

   (c) Based on (a), evaluate $e^{5x}$.

   (d) Evaluate: $\displaystyle\int_1^4 \frac{2^{\sqrt{t}}}{\sqrt{t}} dt$.

   (e) Solve the following equation for $z$: $\ln(10 \cdot \ln z) = \ln(5t^2)$.

7. *A complicated function*

   Let $y(x) = x^{\frac{1}{x-1}}, \ \ x > 0$.

   (a) Evaluate $y(2)$ and $y(3)$.

   (b) Find $\lim_{x \to 1^+} y(x)$, if the limit is finite. If the limit is infinite, state this explicitly. If there is no limit as $x \to 1^+$, explain why.

   (c) Find $\lim_{x \to \infty} y(x)$, if the limit is finite. If the limit is infinite, state this explicitly. If there is no limit as $x \to \infty$, explain why.

   (d) Calculate $\dfrac{dy}{dx}$.

8. *Extra credit: Have a good vacation.*

**Did you mark on the front of your bluebook which problem you want to skip? (Don't skip #8.)**

**Helpful facts:**

Trapezoidal Rule: $T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + ... + 2y_{n-1} + y_n), \ h = \frac{b-a}{n}, \ y_k = f(x_k)$

Error estimate for trapezoidal rule: $|E_T| \leq \dfrac{(b-a)h^2 M}{12}$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \text{ if } u^2 < a^2$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C \text{ if } u^2 > a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$