APPM 1350 – Exam 1
Wednesday, 13 February 2008 A.D.

**INSTRUCTIONS:** Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) student number, (3) instructor's name, and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. This exam has 6 problems, on both sides of this sheet, plus the "Valentine", which can replace any other problem. Work all 6 problems. Start each problem on a new page. Show your work. **BOX** in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (10 points) Consider the functions $f(x) = x - 7$, $g(x) = \sqrt{x}$, $h(x) = \sec(x/2)$.
   (a) Find the domains of $f$, $g$ and $h$.
   (b) Calculate $(f \circ g)(x)$.
   (c) Find its domain and range.

2. (20 points)
   (a) Define the derivative of a function $f$ at a point $x = a$.
   (b) Using the definition at (a), calculate the derivative of $f(x) = \dfrac{x-2}{x}$ at $a = 1$.
   (c) Using (b), find the equation of the tangent line to the graph of $f$ at the point $(1, -1)$.

3. (20 points) Calculate the following limits. If the limit does not exist, write DNE.
   (a) $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$
   (b) $\lim\limits_{s \to -1} \dfrac{\sqrt{s^2 + 8} - 3}{s + 1}$
   (c) $\lim\limits_{h \to 0} \dfrac{h + 2}{h^4 + h^2}$

4. (15 points) On which intervals are the following functions differentiable? Calculate their derivatives (wherever they exist).
   (a) $y = \dfrac{2x + 5}{3x - 2}$
   (b) $y = |x - 5|$

5. (15 points)

   (a) What are the conditions for a function $f$ to be continuous at a point $x = c$?

   (b) For what value of $a$ is the following function continuous at every $x$?

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

6. (20 points) A rock falls from the top of a 100-ft cliff. Its height above the ground after $t$ seconds is $s(t) = 100 - 16t^2$.

   (a) Calculate its average velocity on $[0, 1]$.

   (b) Calculate its instantaneous velocity, its speed and its acceleration at $t = 1$.

   (c) Is the object speeding up or slowing down at $t = 1$?

**Extra credit:** Imagine your heart as the contour given by the union of the graphs of the two functions $y = |x| + \sqrt{1 - x^2}$ (solid line) and $y = |x| - \sqrt{1 - x^2}$ (dashed line), for $-1 \leq x \leq 1$. Cupid always shoots arrows horizontally towards your heart. If he hits it centrally, you fall in love. If he hits it tangentially, you get blue. If he misses, he won't next time. Can you tell at what point(s) on the contour has your heart been hit if you start to feel blue?