

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name (Radulescu or Chang) and (3) recitation number on the front of your bluebook. There are **6 problems**, plus an extra credit problem. Show your work clearly and box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points)

- Estimate the integral $\int_1^2 \frac{1}{t^2} dt$ using $n = 4$ rectangles and left-hand end-points. Do not simplify your numerical answer.
- Estimate the same integral using the trapezoidal sum for $n = 4$. Do not simplify your numerical answer. Hint: $T_n = \frac{h}{2}(y_0 + 2y_1 + \dots + 2y_{n-1} + y_n)$.
- Which of the above is an underestimate/overestimate of the integral?

2. (20 points)

- Calculate the average value of $y = \sqrt{1-x^2}$ on $[0, 1]$. Hint: graph the function and calculate the integral as an area.
- Write the statement of the Mean Value Theorem.
- Find the value of c from the Mean Value Theorem for the function defined in (a).

3. (20 points) Calculate the integrals:

- $\int_1^4 \frac{1 + \sqrt{t}}{t} dt$
- $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2 + 1}} dx$

4. (15 points)

- Calculate the derivative of the function $h(x) = \int_0^{\sqrt{x}} \sin t^2 dt$.
- Find the critical points of $h(x)$.

5. (15 points) Solve the differential equation: $\frac{ds}{dt} = 2 \cos^3 t \sin t$ for the initial value $s(\pi) = 0$.

6. (10 points) Let $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1$. Hint: $f(3) = -1$.

Extra-credit: (20 points)

A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one-third of the sheet on each side through an angle θ . How should θ be chosen so that the gutter will carry the maximum amount of water? You may use the following guidelines:

(a) We will maximize the volume of the rain gutter by maximizing the area of the trapezoid-shaped cross-section $ABCD$ formed by the sides of the gutter. Let h represent DH , the height of the trapezoid, and let b represent AD , the length of the longer base. Let x represent length CH .

(b) Find an expression for h in terms of $\sin \theta$. Find an expression for x in terms of $\cos \theta$. What is b in terms of x ?

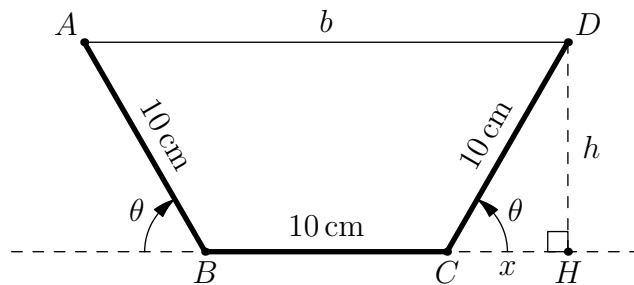
(c) Write an expression for area A in terms of θ .

(Hint: The formula for area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.)

(d) Find $dA/d\theta$ by differentiating A with respect to θ .

(e) Find the maximum area A by finding the critical points of $dA/d\theta$. For what value of θ will the rain gutter hold the maximum amount of water? Express your answer in radians.

(Hint: You may wish to use the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$.)



Good luck!