APPM 1350 – Final Exam
Saturday, 3 May 2008 A.D.

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) student number, (3) instructor's name (Radulescu or Chang), and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 8 problems, plus a total score. **Work all problems from 1 to 6, and your choice of 7 or 8.** Start each problem on a new page. Show your work. **BOX** in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

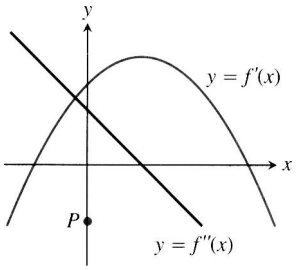
1. (30 points)

   a. Define what it means for a function f to be differentiable at an interior point $x = a$ in its domain.

   b. Suppose $f(x) = \dfrac{1}{x-1}$ and $g(x) = x + 1$. Where is f continuous? Where is g continuous? Where is $(f \circ g)(x)$ continuous?

   c. Give an example of two functions $F(x)$ and $G(x)$, both continuous at $x = 0$, for which the composite $F \circ G$ is discontinuous at $x = 0$.

   d. The graph below shows the first and second derivatives of a function $y = f(x)$. Copy the picture and add to it a sketch of the approximate graph of f, given that the graph passes through the point P. Label any local extrema and inflection points.



2. (30 points) Calculate the following limits:

   a. $\displaystyle\lim_{x\to-\infty} \frac{3-(2/x)}{4+(\sqrt{2}/x^2)} =$

   b. $\displaystyle\lim_{x\to 0} \frac{3^x-1}{2^x-1} =$

   c. $\displaystyle\lim_{x\to 0} \frac{x^2-2x}{x^2-\sin x} =$

3. (20 points)

   a. Calculate the definite integral:

$$\int_{-\pi/2}^{\pi/2} \sin^5(t)dt$$

   Hint: It is not necessary to compute the antiderivative of $\sin^5(t)$ directly.

b. Calculate the indefinite integral:

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

c. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{x/2}$, below by the curve $y = e^{-x/2}$, and on the right by the line $x = 2\ln 2$.

4. (20 points) Consider the function: $y = \left(\frac{1}{x} + 1\right)^x$.

a. Calculate dy/dx.

b. Calculate the limit of y as x approaches ∞.

5. (20 points)

$$f(x) = \tan^{-1}\frac{1}{x} \qquad\qquad g(x) = \cos^{-1}\frac{1}{\sqrt{x^2+1}}$$

a. Evaluate $f(-\sqrt{3})$ and $g(\sqrt{3})$.

b. What is the relationship between $f(x)$ and $g(x)$? Explain.

c. Find the linearization of $f(x)$ at $x = 1$.

6. (10 points) Suppose the amount of oil pumped from a well decreases at the continuous rate of 10% per year. When will the well's output fall to one-fifth of its present value?

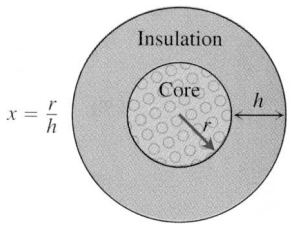
7. (20 points) A 13-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a. How fast is the top of the ladder sliding down the wall then?

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

c. At what rate is the angle θ between the ladder and the ground changing then?

8. (20 points) A round underwater transmission cable consists of a core of copper wires surrounded by nonconducting insulation. If x denotes the ratio of the radius of the core to the thickness of the insulation, it is known that the speed of the transmission signal is given by the equation $v = x^2 \ln(1/x)$. If the radius of the core is 1 cm, what insulation thickness h will allow the greatest transmission speed?



Formulas:

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad\qquad \frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx}$$