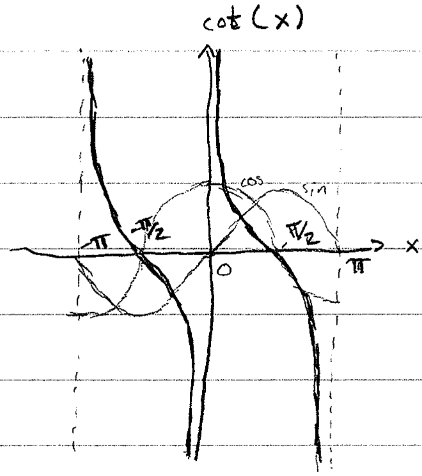
APPM 1350 Exam 1 Solutions

Summer 2008

1) a)



$\cot x = \frac{\cos x}{\sin x}$

b) $x^2 + y^2 - 3y - 4 = 0$

$x^2 + (y^2 - 3y) = 4$

$x^2 + (y^2 - 3y + \frac{9}{4}) - \frac{9}{4} = 4$

$x^2 + (y - \frac{3}{2})^2 = \frac{25}{4}$

center at $(0, \frac{3}{2})$, radius $= \frac{5}{2}$

c) $\cos^2(\theta) = \frac{1 - \cos(2\theta)}{2}$

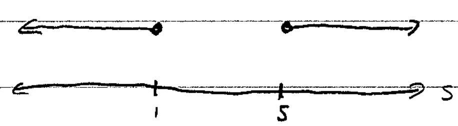
$\cos^2(\frac{1}{3}) = \frac{1 - \cos(2/3)}{2}$

d) $|3 - s| \geq 2$

$3 - s \geq 2$ $\quad\quad$ $3 - s \leq -2$

$-s \geq -1$ $\quad\quad$ $-s \leq -5$

$s \leq 1$ $\quad\quad$ $s \geq 5$



2) a) $g(t) = \frac{t}{4} - t^{5/3} - 42 + \pi^2$

$g'(t) = \frac{1}{4} - \frac{5}{3}t^{2/3}$

b) $f(x) = \left(\frac{x+1}{x^2-4}\right)$

$\frac{df}{dx} = \frac{(x^2-4)(1)-(x+1)(2x)}{(x^2-4)^2}$

c) $y = (x+5)(6x^3-x^2)$

$y' = (x+5)(18x^2-2x) + (6x^3-x^2)(1)$

d) $y = x^2+3x$

$y' = 2x+3$

$\frac{d^2y}{dx^2} = 2$

$y^{(3)} = 0$

3) a) $\lim_{x\to -3} \frac{1}{x} = -\frac{1}{3}$

b) given $\varepsilon > 0$

$\left| \frac{1}{x} - \frac{-1}{3} \right| < \varepsilon$

$\left| \frac{1}{x} + \frac{1}{3} \right| < \varepsilon$

$\left| \frac{x+3}{3x} \right| < \varepsilon$

$|x+3| < |3x|\varepsilon \Rightarrow \boxed{\delta = |3x|\varepsilon}$

4. a) False $\lim_{x\to -1^+} f(x) = 0$

b) False $\lim_{x\to 1} f(x) = 0$

c) True

d) True

e) False $\lim_{x\to 2} f(x) =$ Does not exist

f) False $f'(0) =$ Does not exist

5. a) Car A is ahead after 1 minute because its velocity is always greater than the other two for this period

b) All three cars have the same average acceleration because they all started at zero mph and ended at 60 mph.

c) Car B had constant acceleration because its velocity graph is a straight line (thus constant derivative)

d) Constant acceleration so car B's velocity function is $V = \frac{60\text{ mph}}{60\text{ sec}}(t-0)+0$

so $V(40\text{ sec}) = \frac{60\text{ mph}}{60\text{ sec}} \cdot 40\text{ sec} = 40\text{ mph}$

6. a) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}\left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4} \frac{x-4}{x-4(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$

b) $\lim_{t\to -2} \frac{3t^2+1}{2+t} =$

$(t+2)(at+b) = at^2 + (2a+b)t + 2b$

want $a=3$

$2a+b=0$ $\Rightarrow 2(3)+\frac{1}{2} \neq 0$ so cannot factor

$2b=1 \Rightarrow b=\frac{1}{2}$ out $2+t$ from

numerator

so limit does not exist

(right hand limit is infinity, left hand limit is

negative infinity)

c) $\lim_{\theta\to 1} \frac{3-3\cos\theta}{\theta}$ = (just plug it in) = $3-3\cos(1)$

7. a) True $\cos^2\theta + \sin^2\theta = 1 \Rightarrow$ (divide by $\cos^2\theta$) $1+\tan^2\theta = \sec^2\theta$

b) False $(-1)^5 + (-1)^3 + 1 = -1 + -1 + 1 = -1$

$-(1^5 + 1^3 + 1) = -3$ so $f(-x) \neq -f(x)$

c) False $\frac{\pi}{10}^\circ \cdot \frac{\pi}{180} = \frac{\pi^2}{1800}$ radians $\neq 18$ radians

d) False $\sec\left(-\frac{\pi}{3}\right) = \frac{1}{\cos(-\frac{\pi}{3})} = \frac{1}{\cos(\pi/3)} = \frac{1}{\frac{1}{2}} = 2 \neq -2$

e) False $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = -\lim_{h\to 0} \frac{f(x)-f(x+h)}{h}$, so this is

the definition of $-f'(x)$

8. $S = 2882\,Q^{-1} - 0.052\,Q + 31.73$ for $60 \leq Q \leq 400$

a) $\frac{dS}{dQ} = -2882\,Q^{-2} - 0.052$

b) $\frac{dS}{dQ} = \underbrace{\frac{-2882}{Q^2}} - \underbrace{0.052}_{\text{negative and independent of } Q}$

the numerator is negative, the denominator is alway positive (as $Q \neq 0$) thus the fraction is negative

c) since $\frac{dS}{dQ}$ is always negative this means that as Q increases the speed will decrease and as Q decreases the speed will increase.