

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (20 points) Solve for  $y'$ .

(a)  $\cos(xy) - y = 3x + y^2$

(b)  $y = (3 - \sqrt{x})^{107} + \sec(\pi^2)$

(c)  $y = \sin(\sin(\sin(x^2)))$

(d)  $y = \left(\frac{x+1}{x^2-1}\right)^2$  [Don't Simplify]

2. <sup>18 pts</sup>~~(20 points)~~ When a certain polyatomic gas undergoes adiabatic expansion, its pressure  $p$  and volume  $v$  satisfy the equation  $pv^{1.3} = k$  where  $k$  is a constant.

(a) If the volume is decreasing at a constant rate of  $0.1 \frac{ml}{sec}$ , determine how the pressure is changing the instant the volume is 1 ml. Assume  $k = 10$  and that pressure is measured in atmospheres (atm).

(b) Is the pressure increasing or decreasing?

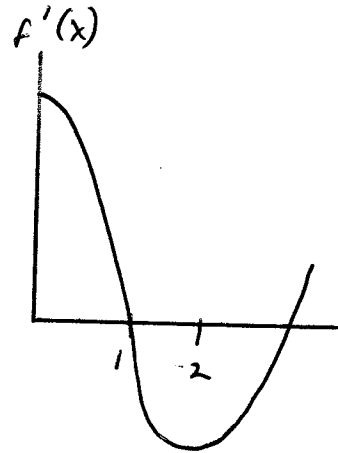
3. <sup>18 pts</sup>~~(20 points)~~ A farmer is going to fence a rectangular pasture where one side is adjacent to a river (thus requiring no fencing).

(a) Draw a picture of the fenced region and label each side.

(b) If the farmer needs  $180,000m^2$  for the pasture, what dimensions will require the least amount of fencing? In order to receive full credit for this question you must justify you have found the extrema by using either the first or second derivative test.

4. (12 points) Use the graph of  $f'(x)$  below to replace the question marks with  $<$ ,  $=$ , or  $>$  as appropriate.

- (a)  $f(0) ? f(1)$       (b)  $f(1) ? f(2)$   
 (c)  $f'(0) ? 0$       (d)  $f'(1) ? 0$   
 (e)  $f''(0) ? 0$       (f)  $f''(2) ? 0$



5. (20 points) Consider the function  $f(x) = x + \frac{16}{x}$

- (a) Find all critical **point(s)**.  
 (b) Find all inflection **point(s)**.  
 (c) On what interval(s) is the function increasing? decreasing?  
 (d) On what interval(s) is the function concave up? concave down?  
 (e) Identify which points (if any) are relative maxima or minima.  
 (f) Now consider the interval  $[1, 8]$ . Is there a global minimum on the interval? If so, find it, if not, state this. Justify your answer.  
 (g) Use the information from parts a – f to sketch  $f(x)$  on the interval  $[-10, 10]$ .

6. (12 points) Given that  $g(5) = -3$ ,  $g'(5) = 6$ ,  $h(5) = 3$ , and  $h'(5) = -2$ , find  $f'(5)$  (if possible) for each of the following. If it is not possible to find  $f'(5)$ , state what additional information is required.

- (a)  $f(x) = g(x)h(x)$       (b)  $f(x) = g(h(x))$   
 (c)  $f(x) = \frac{g(x)}{h(x)}$       (d)  $f(x) = [g(x)]^3$