APPM 1350 Summer 2008

Exam 3 Written Solutions

1. a) $\int \sec^2(t) - t^{-3/5} + \pi \, dt$

$\boxed{= \tan t - \frac{5}{2} t^{2/5} + \pi t + C}$

b) $\int 10t\sqrt{2t+5}\, dt$

$u = 2t+5 \qquad du = 2\,dt \qquad t = \frac{u-5}{2}$

$\Rightarrow 5 \int \left(\frac{u-5}{2}\right) u^{1/2}\, du$

$= \frac{5}{2} \int (u-5) u^{1/2}\, du = 5 \int u^{3/2} - 5u^{1/2}\, du$

$= \frac{5}{2} \left(\frac{2}{5} u^{5/2} - \frac{10}{3} u^{3/2} \right) + C$

$\boxed{= (2t+5)^{5/2} - \frac{25}{3}(2t+5)^{3/2} + C}$

c) $\int_{-3}^{3} r\sqrt{r^2+1}\, dr$

$u = r^2+1 \qquad du = 2r\,dr \qquad u(-3) = (-3)^2+1 = 10$

$u(3) = (3)^2 + 1 = 1$

$\Rightarrow \int_{10}^{10} \frac{1}{2} u^{1/2}\, du \; \boxed{= 0}$ (upper and lower bound is the same)

d) $\frac{d}{dx} \int_5^{x^2} \frac{\ln(z)}{\sqrt{z^4+9}}\, dz \; \boxed{= \frac{\ln(x^2)}{\sqrt{x^8+9}} \cdot 2x}$

e) $\int_{-2}^{-1} \frac{3x^4 - 4x^2}{x^2}\, dx = \int_{-2}^{-1} 3x^2 - 4\, dx$ (since $x=0$ is not in the region)

$= x^3 - 4x \Big|_{-2}^{-1} = [(-1)^3 - 4(-1)] - [(-2)^3 - 4(-2)]$

$= [-1 + 4] - [-8 + 8]$

$= -1 + 4 + 8 - 8 \; \boxed{= 3}$

2. $v(0) = 32 \ \frac{ft}{sec}$ $\qquad s(0) = {}^{-}17 \ ft$

$a = {}^{-}32 \ \frac{ft}{sec^2}$

$v(t) = {}^{-}32t + c \qquad v(0) = c \Rightarrow c = 32$

$v(t) = {}^{-}32t + 32$

$s(t) = {}^{-}16t^2 + 32t + c \qquad s(0) = c \Rightarrow c = {}^{-}17$

$s(t) = -16t^2 + 32t - 17$

now find extrema

$s'(t) = v(t) = -32t + 32$

$v(t) = 0 \Rightarrow t = 1$

$s''(1) = a = {}^{-}32 \Rightarrow$ maximum (local)

$s(1) = -16 + 32 - 17$ $\boxed{= -1 \text{, not high enough to get out of the hole}}$

3. $T(t) = -10\sin\left(\frac{\pi t}{12}\right) + 15$

find average temperature $\Rightarrow$ Mean Value Theorem

$$\text{average temp} = \frac{1}{24-0}\int_0^{24} -10\sin\left(\frac{\pi t}{12}\right) + 15 \, dt$$
$$= \frac{1}{24}\left[\int_0^{24} -10\sin\left(\frac{\pi t}{12}\right)dt + \int_0^{24} 15 \, dt\right]$$
$$= \frac{1}{24}\left[10\left(\frac{12}{\pi}\right)\cos\left(\frac{\pi t}{12}\right)\Big|_0^{24} + 15t\Big|_0^{24}\right]$$

$$= \frac{1}{24}\left[\left(\frac{120}{\pi}(1) - \frac{120}{\pi}(1)\right) + \left(15(24) - 15(0)\right)\right] = \frac{1}{24}\left(15(24)\right)$$
$$= \frac{1}{24}(360) =$$

$$\boxed{= 15\,^\circ F}$$

4. a) $f(x) = \sqrt{x}$ $\qquad f'(x) = \frac{1}{2\sqrt{x}}$

linearization at some x_0 is

$L(x) = f'(x_0)(x - x_0) + f(x_0)$

to make algebra easy, pick $x_0 = 9$ and 16

(the nearest perfect squares to 12)

$L_1(x) = f'(9)(x-9) + f(9) \boxed{= \frac{1}{6}(x-9) + 3}$

$L_2(x) = f'(16)(x-16) + f(16) \boxed{= \frac{1}{8}(x-16) + 4}$

b) $L_1(12) = \frac{1}{6}(12-9) + 3 = \frac{1}{6}(3) + 3 \boxed{= 3.5}$

$L_2(12) = \frac{1}{8}(12-16) + 4 = \frac{1}{8}(-4) + 4 \boxed{= 3.5}$

c) Newton's Method: $x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$

$g(x) = x^2 - 12 \qquad g'(x) = 2x$

$x_0 = 4$

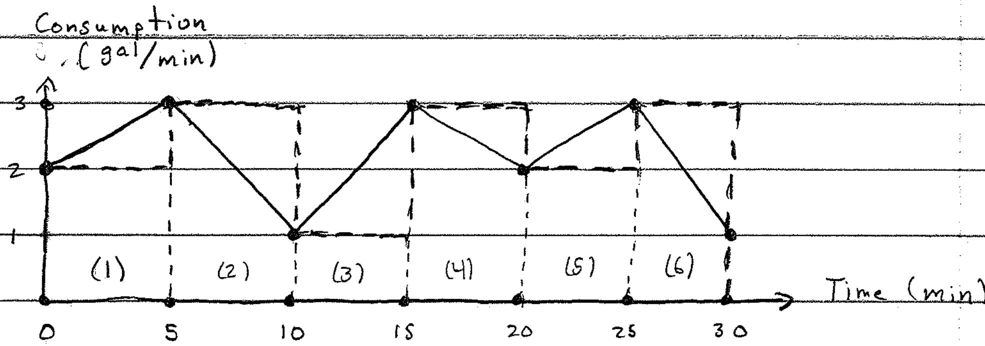
$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = 4 - \frac{4}{8} = 3.5 \boxed{= \frac{7}{2}}$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = \frac{7}{2} - \frac{\left(\frac{7}{2}\right)^2 - 12}{2\left(\frac{7}{2}\right)}$

$= \frac{7}{2} - \frac{\frac{49}{4} - \frac{48}{4}}{7} = \frac{49}{14} - \frac{2\left(\frac{1}{4}\right)}{14}$

$= \frac{98}{28} - \frac{1}{28} \boxed{= \frac{97}{28}}$

5. a)



b) dashed lines represent the left endpoint approximation method with 6 rectangles

c) total consumption $\approx 5(2+3+1+3+2+3)$
$= 5(14) \boxed{= 70 \text{ gallons}}$

d) This is an over estimate. The over estimate from rectangles (2) and (4) are exactly canceled out by the under estimate from rectangles (3) and (5) respectively. Thus the remaining over estimate from rectangle (6) is larger than the under estimate from rectangle (1) so they do not cancel leaving an overall over estimate