Name: _____

APPM 1350 EXAM #1 Summer 2008

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (16 points) Preliminaries
   (a) Sketch $\cot(x)$ and include a scale on the x-axis.
   (b) Identify the center and the radius of the circle $x^2 + y^2 - 3y - 4 = 0$.
   (c) Use the double angle formula to rewrite $\cos^2\left(\frac{1}{3}\right)$ without any squared terms.
   (d) Solve $|3 - s| \geq 2$ and graph the solution set on the real number line.

2. (16 points) For each function, find the indicated derivative(s). Do **not** simplify your answers.

   (a) $g(t) = \frac{t}{4} - \sqrt[3]{t^5} - 42 + \pi^2$; $g'(t)$ (b) $f(x) = \left(\frac{x+1}{x^2-4}\right)$; $\frac{df}{dx}$

   (c) $y = (x+5)(6x^3 - x^2)$; y' (d) $y = x^2 + 3x$; y', $\frac{d^2y}{dx^2}$, $y^{(3)}$

3. (8 points) The formal definition of a limit states:
   For a function f defined in some open interval containing x_0 (but not necessarily at x_0 itself), we say $\lim_{x \to x_0} f(x) = L$, if for every number $\epsilon > 0$ there exists a corresponding number $\delta > 0$, such that for all x
   $0 < |x - x_0| < \delta \Longrightarrow |f(x) - L| < \epsilon$

   (a) Find $\lim_{x \to -3} \frac{1}{x}$
   (b) Prove the limit exists by finding δ. Leave your final answer in terms of x if applicable.

4. (16 points) The function $f(x)$ is sketched here for $-1 < x < 3$. For each statement, mark **True** if it is true. If it is false you need to do **two things**: 1. mark **FALSE** and 2. correct the right hand side of the equation.

(a) $\lim_{x \to -1^+} f(x) =$ Does Not Exist

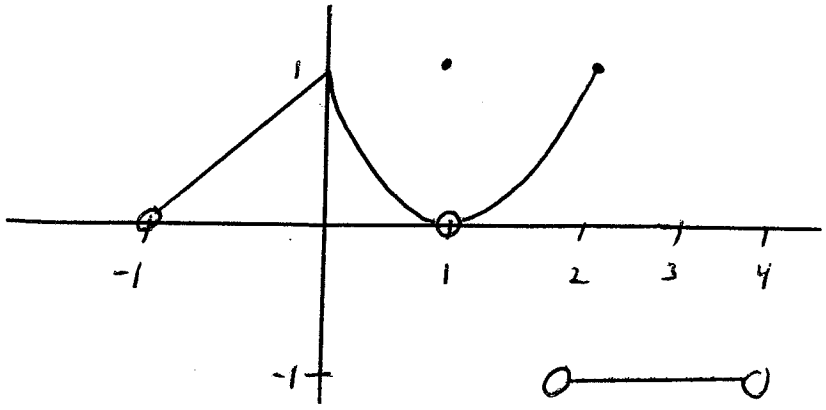
(b) $\lim_{x \to 1} f(x) = 1$

(c) $f(x)$ is continuous at $x = 0$

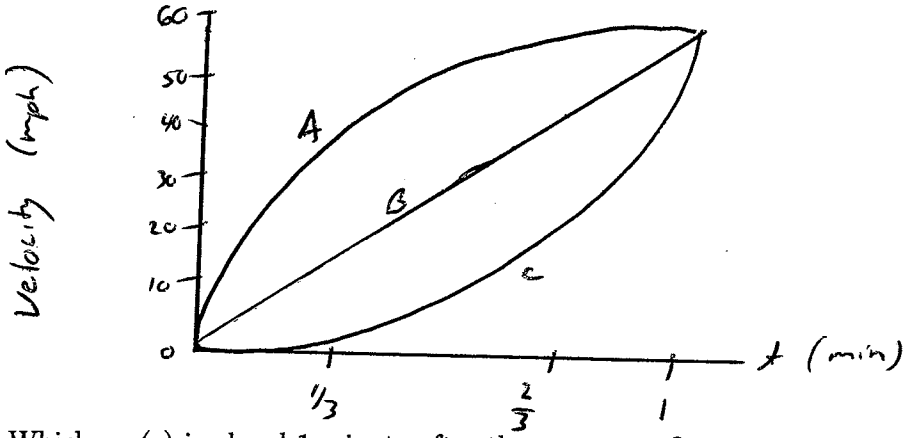
(d) $f(2) = 1$

(e) $\lim_{x \to 2} f(x) = 1$

(f) $f'(0) = 1$



5. (12 points) Three cars start a race at the same time. Based on the graph below (which shows **velocity** in miles per hour versus time in minutes) answer the following questions. You must justify your answer in order to receive full credit.



(a) Which car(s) is ahead 1 minute after the race starts?

(b) Which car(s) had the fastest average acceleration over the first minute?

(c) Which car(s) (if any) had a constant acceleration?

(d) What was the instantaneous velocity of car B at 40 seconds?

6. (12 points) Find the following limits. Give your answers as: a number, $-\infty$, ∞, or DNE (Does Not Exist). You must justify your answer (algebra, graph, a few words, etc) in order to receive full credit for each limit.

(a) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4}$ (b) $\lim_{t\to -2} \dfrac{3t^2+1}{2+t}$ (c) $\lim_{\theta\to 1} \dfrac{3-3\cos(\theta)}{\theta}$

7. (10 points) True or False? Write true if the statement is true, write false if the statement is false. No justification is needed.

(a) $\tan^2\theta + 1 = \sec^2\theta$

(b) $x^5 + x^3 + 1$ is an odd function.

(c) $\dfrac{\pi}{10}^\circ$ is equivalent to 18 radians.

(d) $\sec(\frac{-\pi}{3}) = -2$

(e) The formal definition of derivative states $\lim_{h\to 0} \dfrac{f(x)-f(x+h)}{h}$

8. (10 points) According to a 1997 study, the traffic speed, S, on a two-lane road is related to the traffic density, Q, (number of cars per mile of road) by the formula $S = 2882Q^{-1} - 0.052Q + 31.73$ for $60 \le Q \le 400$

(a) Calculate $\dfrac{dS}{dQ}$.

(b) $\dfrac{dS}{dQ}$ is always negative. Demonstrate this by showing each term in $\dfrac{dS}{dQ}$ is negative.

(c) Explain what happens to the speed as Q increases and what happens to the speed when Q decreases.