APPM 1350 Summer 2008

Exam #2 Solutions

1. a) $\cos(xy) - y = 3x + y^2$

$-\sin(xy)(y + xy') - y' = 3 + 2yy'$

$-2yy' + \sin(xy)xy' - y' = 3 + \sin(xy)y$

$y'(-2y - \sin(xy)x - 1) = 3 + \sin(xy)y$

$y' = \dfrac{3+\sin(xy)y}{-2y - \sin(xy)x - 1}$

b) $y = (3 - \sqrt{x})^{107} + \sec(\pi^2)$

$y' = 107(3-\sqrt{x})^{106} \cdot \frac{-1}{2}x^{-1/2}$

$y' = \dfrac{-107(3-\sqrt{x})^{106}}{2\sqrt{x}}$

c) $y = \sin(\sin(\sin(x^2)))$

$y' = \cos(\sin(\sin(x^2)) \cdot \cos(\sin(x^2)) \cdot \cos(x^2) \cdot 2x$

d) $y = \left(\frac{x+1}{x^2-1}\right)^2$

$y' = 2\left(\frac{x+1}{x^2-1}\right) \cdot \dfrac{(x^2-1)-(x+1)\cdot 2x}{(x^2-1)^2}$

or

$y = \left(\frac{x+1}{x^2-1}\right)^2 = \left(\frac{\cancel{x+1}}{\cancel{(x+1)}(x-1)}\right)^2 = \frac{1}{(x-1)^2} = (x-1)^{-2}$

$y' = -2(x-1)^{-3} \cdot 1$

$= -2(x-1)^{-3}$

2. $pv^{1.3} = k \Rightarrow p = kv^{-1.3}$

a) $\frac{dv}{dt} = {}^{-}0.1 \frac{ml}{sec}$ $\quad V = 1\ ml \quad k = 10$

differentiate $pv^{1.3} = k$ with respect to time

$p' v^{1.3} + 1.3 p v^{0.3} v' = 0$ now plug in

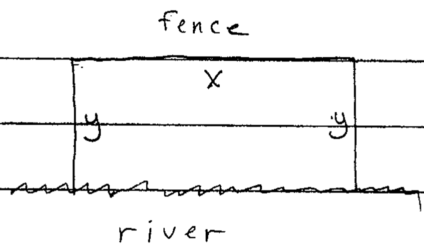
$p'(1)^{1.3} + 1.3(10 \cdot 1^{-1.3})(1)^{0.3} \cdot {}^{-}(0.1) = 0$

$p' + 13 \cdot {}^{-}0.1 = 0$

$p' = 0 + 1.3 = 1.3\ {}^{atm}/_{sec}$ (positive)

b) p' is positive so the pressure is increasing

3. a)



b) area $= 180{,}000\ m^3 = xy$

amount of fence $= x + 2y$ ← minimize this quantity

$180{,}000 = xy \Rightarrow x = 180{,}000\, y^{-1}$

so $a_f = 180{,}000\, y^{-1} + 2y \qquad 0 < y < \infty$

$a_f' = -180{,}000\, y^{-2} + 2$ set equal to zero

$180{,}000\, y^{-2} = 2$

$y^2 = 90{,}000$

$\boxed{y = 300\ m}$ (y is positive) $\Rightarrow \boxed{x = \frac{180{,}000}{300} = 600\,m}$

minima or maxima? second derivative test

$a_f'' = 360{,}000\, y^{3} = 360{,}000\,(300)^3$, positive, so minima

4. a) $f(0) < f(1)$ — f' is positive between 0 and 1 so f is increasing

b) $f(1) > f(2)$ — f' is negative between 1 and 2 so f is decreasing

c) $f'(0) > 0$ — obvious from graph

d) $f'(1) = 0$ — obvious from graph

e) $f''(0) \leq 0$ — tangent to $f'(0)$ has *slightly* negative slope (could be zero)

f) $f''(2) = 0$ — tangent to $f'(2)$ has zero slope

5. $f(x) = x + 16x^{-1}$

$f'(x) = 1 - 16x^{-2}$

$f''(x) = 32x^{-3}$

Note: Since $f(0)$ does not exist there can be no critical or inflection *points* there

a) $0 = 1 - 16x^{-2} \Rightarrow 16x^{-2} = 1 \Rightarrow x^2 = 16$

($f'(0)$ does not exist)

so critical points at $x = \pm 4 \Rightarrow (-4, -8)$ and $(4, 8)$

b) $0 = 32x^{-3} \Rightarrow x^{-3} = 0$, not solvable, no inflection points

($f''(0)$ does not exist)

c) increasing when $f'(x)$ is positive $\Rightarrow (-\infty, -4) \cup (4, \infty)$

decreasing when $f'(x)$ is negative $\Rightarrow (-4, 0) \cup (0, 4)$

d) concave up when $f''(x)$ is positive $\Rightarrow (0, \infty)$

concave down when $f''(x)$ is negative $\Rightarrow (-\infty, 0)$

5\. e) $x = \pm 4$ are critical points, use second derivative test

$f''(-4) = 32(-4)^{-3}$ is negative

so $(-4,-8)$ is a relative max

$f''(4) = 32(4)^{-3}$ is positive

so $(4, 8)$ is a relative min

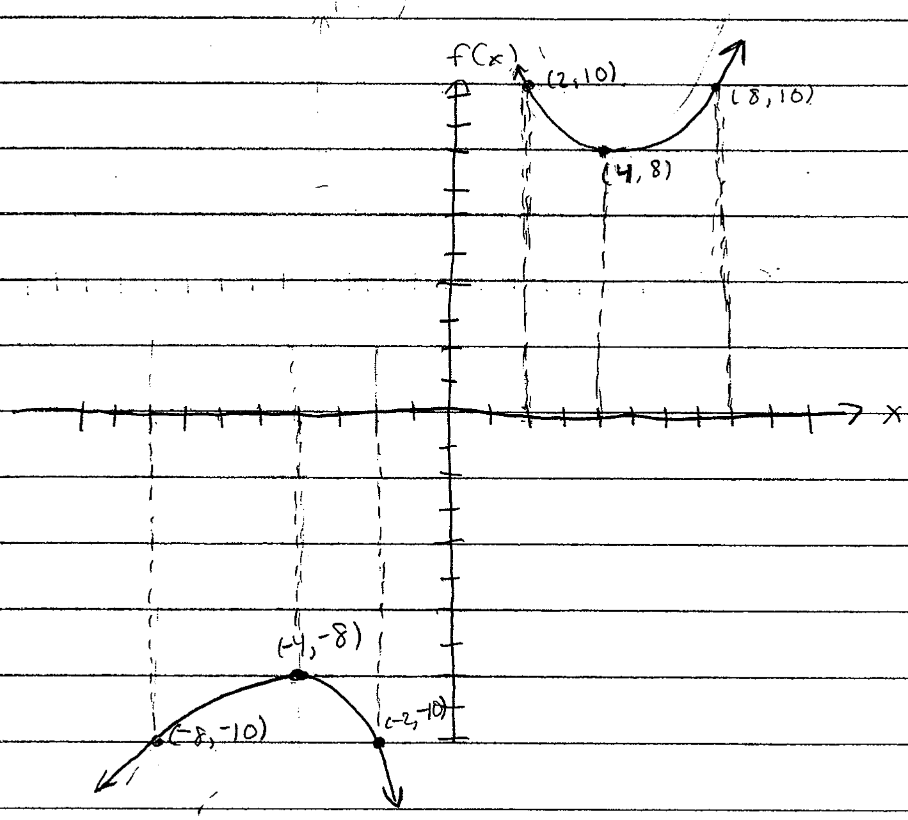
f) check end points and critical points in interval (Note: $f(x)$ is continuous and differentiable in this interval)

$f(1) = 1 + 16 = 17$

$f(4) = 8$ so $(4,8)$ is global min

$f(8) = 8 + 2 = 10$

g)



6. $g(5) = -3 \qquad g'(5) = 6$

$h(5) = 3 \qquad h'(5) = -2$

Find $f'(5)$ for

a) $f(x) = g(x)h(x)$

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(5) = g'(5)h(5) + g(5)h'(5)$

$= 6 \cdot 3 + {}^{-}3 \cdot {}^{-}2 = 24$

b) $f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$f'(5) = g'(h(5)) \cdot h'(5)$

$= g'(3) \cdot {}^{-}2$, need $g'(3)$ to solve

c) $f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{h^2(x)}$

$f'(5) = \frac{h(5)g'(5) - g(5)h'(5)}{h^2(5)}$

$= \frac{3 \cdot 6 - {}^{-}3 \cdot {}^{-}2}{3^2} = \frac{18-6}{9} = \frac{12}{9} = \frac{4}{3}$

d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 \cdot g'(x)$

$f'(5) = 3[g(5)]^2 \cdot g'(5)$

$= 3(-3)^2 \cdot 6 = 3 \cdot 9 \cdot 6 = 3 \cdot 54 = 162$