APPM 1350 Final Exam Summer 2008

Be sure to include your name and a grading table on the front of your blue book. You must work the first three problems and 2 of the last 4 questions. Note that this exam is worth 150 points. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please start each problem **on a new page**. Good luck!

1. (40 points) Work 5 of the following 6 integrals.

   (a) $\int \sqrt{\theta} + e^{2\theta} + \pi \, d\theta$

   (b) $\int_0^2 \frac{x}{x^2 - 5} \, dx$

   (c) $\int \sin^2(r) \, dr$

   (d) $\int \frac{z}{\sqrt{z^2 + 9}} \, dz$

   (e) $\int_0^1 \frac{\arcsin(x)}{\sqrt{1 - x^2}} \, dx$

   (f) $\int \frac{\sin(\theta)}{1 + \cos^2(\theta)} \, d\theta$

2. (32 points) Find $\frac{dy}{dx}$ for 4 of the following 5 derivatives.

   (a) $y = \ln(3xe^{2x}(2x - 1)^6))$

   (b) $y = \pi^x + \log_3(x^5)$

   (c) $y = x \sin^{-1}(x) + \sqrt{1 - x^2}$

   (d) $y = \frac{\csc(x)}{2x + 0.5}$

   (e) $y = e^{xy}$ [Do not simplify]

3. (24 points) Evaluate the following limits. If the limit doesn't exist or is infinite please indicate either ∞, $-\infty$, or DNE. You must justify your answer (math, words, etc)to receive full credit.

   (a) $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$

   (b) $\lim_{x \to -\infty} \frac{7844x^{13} + 400x^{12}}{0.000005x^{14}}$

   (c) $\lim_{x \to 0} \frac{x - \sin(x)}{x^3}$

WORK 2 OF THE NEXT 4 PROBLEMS

4. (27 points) A 13 foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of $10 \frac{ft}{sec}$ away from the wall. Find the velocity at which the top of the ladder is sliding down the wall at that point.

5. (27 points) The following 9 pieces describe different parts of the **same** *continuous* function $f(x)$. Sketch $f(x)$ **and identify** each piece on the graph. Make the graph large and label the axes.

(a) $f(1) = 2$	(b) $\lim_{x\to\infty} f(x) = 6$	(c) $\lim_{x\to-\infty} f(x) = 6$
(d) $f'(x) < 0$ for $x < 1$	(e) $f'(x) > 0$ for $x > 1$	(f) $\lim_{x\to1^+} f'(x) = \infty$
(g) $\lim_{x\to1^-} f'(x) = -\infty$	(h) $f''(x) > 0$ for $3 < x < 4$	(i) $f''(x) < 0$ for $x > 4$ and $1 < x < 3$

6. (27 points) Newton's Law of Cooling states that the rate at which an object cools (or heats) is proportional to the difference in temperature between the object (T) and its surroundings (T_s).

   (a) Set up the differential equation for this using k as the constant of proportionality rather than $-k$.

   (b) Solve the differential equation for $T(t)$ in terms of T_s and T_0

   (c) In 1994 a woman sued McDonald's for (among other things) her coffee being too hot. After class you decide to find out the temperature that McDonald's serves coffee at. You drive straight to the McDonald's drive-through, buy a cup of coffee, and 20 minutes later (once you get home) you record a temperature of 130°F. Assume it is a constant 80°F both outside and inside your car (since your car windows are down). Also assume that after 10 more minutes the temperature of the coffee is 120°F. Solve for k.

   (d) Set up but do not evaluate the equation for finding the temperature of the coffee when it was served to you.

7. (27 points) Coughing is achieved by changing the radius of the windpipe to increase the flow of air to the lungs. The relationship between the velocity of the airflow and the radius is given as $V(r) = cr^2(r_0 - r)$ where c is a constant, and r_0 is the *constant* radius of the windpipe at rest.

   (a) Show that $r = \frac{2}{3}r_0$ is a critical point.

   (b) Letting $c = 2$ and $r_0 = 3$, use *either* the first or second derivative test to determine whether your answer in part a is a maximum or minimum.

   (c) According to your results in parts a and b, what happens to the radius of the windpipe when you cough? What happens to the velocity of airflow when you cough?

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{du}{a^2 + u^2} = \left(\frac{1}{a}\right)\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{a^2 + u^2}} = \left(\frac{1}{a}\right)\sec^{-1}\left|\frac{u}{a}\right| + C$$