

A PPM 1350 - Forgiveness Exam 1 - Answers

1. a) $y = \frac{1}{3x^2} - \frac{5}{2x}$

[HW: p. 127, #16]

$$\boxed{\frac{dy}{dx} = \frac{-2}{3x^3} + \frac{5}{2x^2}}$$

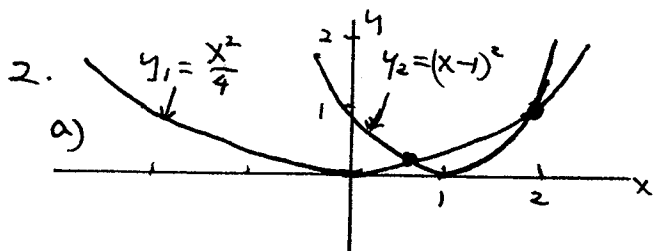
$$\boxed{\frac{d^2y}{dx^2} = \frac{2}{x^4} - \frac{5}{x^3}}$$

b) $y = \frac{x-1}{2x-7}$

$$\frac{dy}{dx} = \frac{(2x-7)(1) - (x-1)(2)}{(2x-7)^2} = \boxed{\frac{-5}{(2x-7)^2}}$$

At $x=1$

$$\boxed{\left. \frac{dy}{dx} \right|_{x=1} = -\frac{1}{5}}$$



b) $y_1(x) = y_2(x)$

[HW: p. 34, #80]

$$\Rightarrow \frac{x^2}{4} = x^2 - 2x + 1$$

$$\Rightarrow \boxed{x=2} \quad \& \quad \boxed{x=\frac{3}{2}}$$

$$\quad \quad \quad \boxed{y=1} \quad \quad \quad \boxed{y=\frac{1}{2}}$$

c) At $x=2$, $y_1(2) = 1$, $\frac{dy_1}{dx} = \frac{x}{2} \Rightarrow$ at $x=2$, $\frac{dy_1}{dx} = 1$

Tangent line: $\boxed{y-1 = (1)(x-2)}$ or $\boxed{y = x-1}$

3. a) T

b) F

$\lim_{x \rightarrow 1} f(x) = 2$

c) T

d) T

e) F

$f(x)$ is discontinuous at $x=1$

4. a) $f(x)$ is continuous at $x=c$ if:

- $\lim_{x \rightarrow c} f(x)$ exists
- $f(c)$ is defined
- $\lim_{x \rightarrow c} f(x) = f(c)$

b) $g(x) = \frac{2x^3 - 9x + 5}{x^2 + 1}$, $-2 < x < 2$.

$g(x)$ is continuous at

every x in $-2 < x < 2$ because

- $g(x)$ is a ratio of polynomials
- polynomials are continuous everywhere

• $x^2 + 1 \geq 1$ so denom $\neq 0$

(OVER)

$$4c) \quad g(0) = 5 > 0, \quad g(1) = -1 < 0$$

By Intermediate Value Theorem, there is a c , $0 < c < 1$
where $g(c) = 0$.

5. Position of rock: $s(t) = 24t - \frac{9.8t^2}{2} \text{ m}$ [HW: p. 140, #1]

a) velocity: $\frac{ds}{dt} = 24 - 9.8t \text{ m/sec}$

acceleration: $\frac{d^2s}{dt^2} = -9.8 \text{ m/sec}^2$

b) Highest point in trajectory occurs at \hat{t} , when

$$\frac{ds}{dt}(\hat{t}) = 24 - 9.8\hat{t} = 0 \Rightarrow \hat{t} = \frac{24}{9.8} \text{ sec}$$

c) Height at \hat{t} : $s(\hat{t}) = 24\left(\frac{24}{9.8}\right) - \frac{9.8}{2}\left(\frac{24}{9.8}\right)^2 = \frac{(24)^2}{19.6} \text{ m}$

d) Rock is aloft for $2\hat{t} = \frac{24}{4.9} \text{ sec}$