

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation section on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. Work all 6 **problems**. Start each problem on a **new page**. Show your work. **Box** in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive parital credit.

1. (20 points) Perform the indicated operations:

a) $\lim_{x \rightarrow 0} \sec[\cos x + \pi \tan\left(\frac{\pi}{4 \sec x}\right) - 1]$

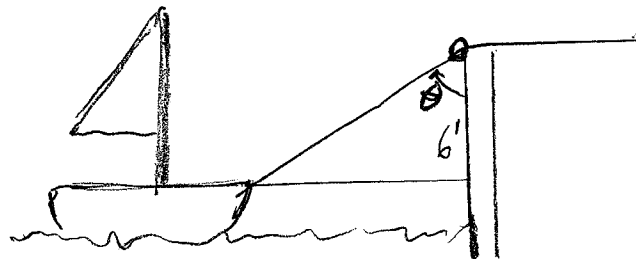
b) If $x^3 + y^3 = 16$,

i. find the value of d^2y/d^2x at the point (2,2)

ii. discuss the concavity of the curve at the point (2,2)

2. (20 points)

A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 feet above the bow. The rope is hauled in at the rate of 2 ft/sec.



- How fast is the boat approaching the dock when 10 ft of rope are out?
- At what rate is angle θ changing then?

Be sure to include the correct units.

3. (20 points)

a) Sketch the graph of the function $f(x) = \frac{x^3 + 2}{2x^2}$. Be sure to identify (and label on the graph) any maxima, minima or inflection points and any asymptotes.

b) Identify the interval(s) on which $f(x)$ is increasing and/or decreasing

c) Identify the interval(s) on which $f(x)$ is concave up and/or concave down

4. (12 points)

If the following statement is ALWAYS TRUE write T or if the statement is NOT NECESSARILY TRUE write F. If the statement is false, explain why or give a counterexample.

- a) If a function, $f(x)$, is continuous on a closed interval $[a,b]$, then there is at least one point c in $[a, b]$ where $\frac{f(b) - f(a)}{b - a} = f'(c)$
- b) If $f'(x) = g'(x)$ for all real numbers, then $f(x)$ must be equal to $g(x)$.
- c) If a function, $f(x)$, is continuous on an interval and a and b are any two points on that interval, then if y_0 is between $f(a)$ and $f(b)$, there exists a number c between a and b such that $f(c) = y_0$.
- d) If a function, $f(x)$, is differentiable for all real numbers, then its extrema will be found only at x values where $f'(x) = 0$.

5. (16 points)

Find the largest possible value of $s = 2x + y$, if x and y are side lengths in a right triangle whose hypotenuse is $\sqrt{5}$ meters long.

6. (12 points)

- a) Use one iteration of Newton's Method to find x_1 for the function $f(x) = x^3 + 5x - 5$ if $x_0 = 1$.
- b) Copy the graph below of $g(x)$ in your blue book and show where x_1 is, given $x_0 = 4$.