APPM 1350 - Exam 2 Answers

1. a) $\lim_{x\to 0} \sec\left[\cos x + \pi \tan\left(\frac{\pi}{4\sec x}\right) - 1\right]$ $\boxed{\text{HW: p. 152, \#29}}$

All the functions are continuous at the relevant points.
Use: $\cos(0) = \sec(0) = 1$, $\tan(\frac{\pi}{4}) = 1$, $\sec(\pi) = -1 \Rightarrow$ $\boxed{\text{Answer: } -1}$

b) $x^3 + y^3 = 16$ — Find the value of $\frac{d^2y}{dx^2}$ at $(2,2)$ $\boxed{\text{HW: p. 170, \#43}}$

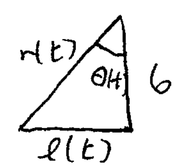
$3x^2 + 3y^2 \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3x^2}{3y^2} = -\frac{x^2}{y^2}$ (implicit differentiation)

$$\frac{d^2y}{dx^2} = \frac{(y^2)(-2x) - (-x^2)(2y\frac{dy}{dx})}{y^4} = \frac{-2xy^2 - 2x^2y\left(\frac{x^2}{y^2}\right)}{y^4} = \frac{-2xy^3 - 2x^4}{y^5}$$

$\boxed{\text{At } (x=2, y=2),\ \frac{d^2y}{dx^2} = \frac{-32-32}{32} = -2}$ Because $\frac{d^2y}{dx^2} < 0$ at $(2,2)$

$\boxed{\text{the curve is } \underline{\text{concave down}} \text{ at } (2,2)}$

2. a) $r^2(t) = \ell^2(t) + 36$ $\boxed{\text{HW: p. 178, \#22}}$



$2r\frac{dr}{dt} = 2\ell\frac{d\ell}{dt}$.

when $r = 10$, $\ell = 8$, so $(2)(10)(-2) = (2)(8)\frac{d\ell}{dt} \Rightarrow$ $\boxed{\frac{d\ell}{dt} = -2.5 \text{ ft/sec}}$

b) $\cos(\theta(t)) = \frac{6}{r(t)}$

$-\sin\theta \frac{d\theta}{dt} = -\frac{6}{r^2}\frac{dr}{dt}$, so $\frac{d\theta}{dt} = \frac{1}{\sin\theta} \cdot \frac{6}{r^2}\frac{dr}{dt}$

when $r = 10$, $\sin\theta = \frac{8}{10} = \frac{4}{5} \Rightarrow$ $\boxed{\frac{d\theta}{dt} = \frac{1}{(\frac{4}{5})} \cdot \frac{6}{100}(-2) = -\frac{3}{20} \text{ rad/sec}}$

3. $f(x) = \frac{x^3+2}{2x^2} = \frac{x}{2} + \frac{1}{x^2} \Rightarrow$ $f(x)$ has an $\underline{\text{oblique asymptote: } y = \frac{x}{2}}$

and a $\underline{\text{vertical asymptote: } x = 0}$

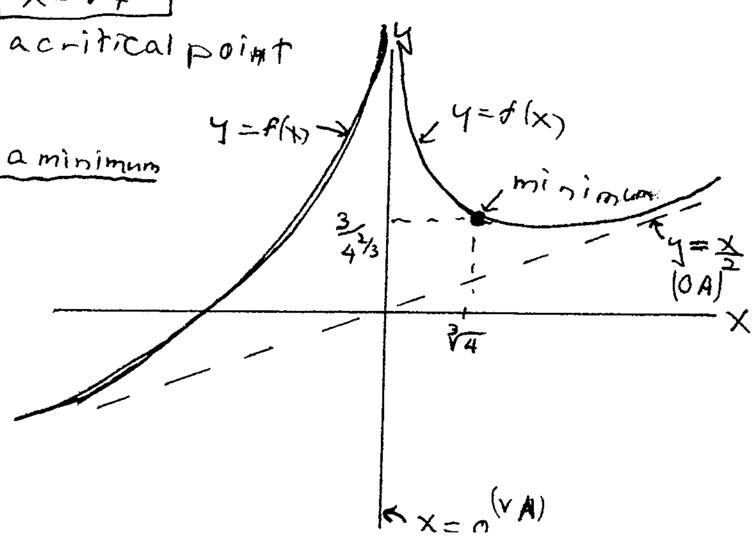
$\frac{df}{dx} = \frac{1}{2} - \frac{2}{x^3}$, so $\frac{df}{dx} = 0 \Rightarrow x^3 = 4 \Rightarrow$ $\boxed{x = \sqrt[3]{4}}$ is a critical point

- At $x = \sqrt[3]{4}$, $f = 3/4^{2/3}$
- For $x < 0$, $\frac{df}{dx} > 0$. For $0 < x < \sqrt[3]{4}$, $\frac{df}{dx} < 0$.
  For $x > \sqrt[3]{4}$, $\frac{df}{dx} > 0 \Rightarrow$ $\underline{(x = \sqrt[3]{4}, y = 3/4^{2/3}) \text{ is a minimum}}$

$\frac{d^2f}{dx^2} = \frac{6}{x^4} > 0 \Rightarrow$ curve is $\underline{\text{always}}$ $\underline{\text{concave } \cancel{\text{down}} \text{ up}}$, & the critical point is a $\underline{\text{local minimum}}$

$f(x)$ decreasing on $(0, \sqrt[3]{4})$

$f(x)$ increasing on $(-\infty, 0)$ and $(\sqrt[3]{4}, \infty)$



4 a) $\boxed{F}$ $f(x)$ must be differentiable to use the Mean-Value Theorem. A counter example is $f(x) = |x|$, $a=-1$, $b=+1$.

b) $\boxed{F}$ $f'(x) = g'(x) \Rightarrow f(x) = g(x) + C$, for $\underline{\text{any}}$ constant C.

c) $\boxed{T}$ Int. Value Th..

d) $\boxed{T}$ As stated, $f(x)$ is differentiable everywhere, and it has no endpoints

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5. Maximize $s = 2x + y$
subject to $x^2 + y^2 = 5$

① Endpoints: $x = 0,\ y = \sqrt{5},\ s = \sqrt{5}$ m
$x = \sqrt{5},\ y = 0,\ s = 2\sqrt{5}$ m

② Otherwise, $y = \sqrt{5 - x^2}$, so $s(x) = 2x + \sqrt{5 - x^2}$

$$\frac{ds}{dx} = 2 + \frac{(-2x)}{2\sqrt{5 - x^2}} = 2 - \frac{x}{\sqrt{5 - x^2}}. \quad \text{So}$$

③ $$\frac{ds}{dx} = 0 \Rightarrow x = 2\sqrt{5 - x^2} \Rightarrow x^2 = 4(5 - x^2) \Rightarrow 5x^2 = 20 \Rightarrow \boxed{x = 2}$$

$x = 2$ is the $\underline{\text{only}}$ critical point with $0 < x < \sqrt{5}$

$\boxed{\text{At } x = 2\text{m}, y = 1\text{m}, s = 2(2) + 1 = 5\text{m} = s}$ Note that $5 > 2\sqrt{5} > \sqrt{5}$

④ $$\frac{d^2 s}{dx^2} = \frac{\sqrt{5 - x^2}(-1) - (-x)\left(\frac{-2x}{2\sqrt{5 - x^2}}\right)}{(5 - x^2)} = \frac{-(5 - x^2) - x^2}{(5 - x^2)^{3/2}} = \frac{-5}{(5 - x^2)^{3/2}} < 0$$

$\frac{d^2 s}{dx^2} < 0 \Rightarrow \boxed{x = 2,\ s = 5\text{m} \text{ is a maximum}}$

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6. a) $f(x) = x^3 + 5x - 5$, $f'(x) = 3x^2 + 5$

Newton: $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$ At $x_0 = 1$, $f(x_0) = 1$, $f'(x_0) = 8$

$x_1 = 1 - \frac{1}{8} = \boxed{7/8 = 0.875 = x_1}$

b)