

APPM FORGIVENESS EXAM 2 - ANSWER KEY

1. a) $\frac{df}{dt} = \frac{1}{2} (1 + \cos^2(7t))^2 \cdot 2 \cos(7t) (-\sin(7t)) \cdot 7$

$= -7(1 + \cos^2(7t))^2 \cos(7t) \sin(7t)$

b) $18z^2 y \frac{dz}{dy} + 6z^3 + 4 = 0$

$\frac{dz}{dy} = \frac{6z^3 + 4}{-18z^2 y} = \frac{3z^3 + 2}{-9z^2 y}$

$\frac{dz}{dy} = \frac{-9z^2 y \left(9z^2 \frac{dz}{dy}\right) + (3z^3 + 2)(9z^2 + 18z y \frac{dz}{dy})}{81z^4 y^2}$

$= \frac{-81z^4 \left(\frac{3z^3 + 2}{-9z^2 y}\right) + (3z^3 + 2)(9z^2 + 18z y \left(\frac{3z^3 + 2}{-9z^2 y}\right))}{81z^4 y^2}$

$= \frac{-81z^4 (3z^3 + 2) + (3z^3 + 2) [-81z^4 y + (54z^4 y + 36z y)]}{-729 z^6 y^3}$

$= \frac{(3z^3 + 2) [-81z^4 - 27z^4 y + 36z y]}{-729 z^6 y^3}$

2. a) $V = X^3$

$\frac{dV}{dX} = 3X^2 \Rightarrow dV = 3X^2 dX$

$dV \Big|_{X=10}^{X=11} = 300(1.1) = 330 \text{ cu in}$

b) $y = \sin x$
 $y' = \cos x$

$y'(x) = -1$

$y(x) = 0$
 $y - 0 = -1(x - \pi)$
 $y = -x + \pi$

$\sin 3 \approx -3 + \pi$

3. $4x + 2\pi r = 4$
 $2x + \pi r = 2$
 $x = \frac{2 - \pi r}{2}$

MAX: $A = \pi r^2 + X^2$

$A = \pi r^2 + \left(\frac{2 - \pi r}{2}\right)^2 = \pi r^2 + \frac{4 - 4\pi r + \pi^2 r^2}{4}$
 $= \pi r^2 + 1 - \pi r + \frac{\pi}{4} r^2$

$A' = 2\pi r - \pi + \frac{\pi}{2} r$

$2\pi r - \pi + \frac{\pi}{2} r = 0$

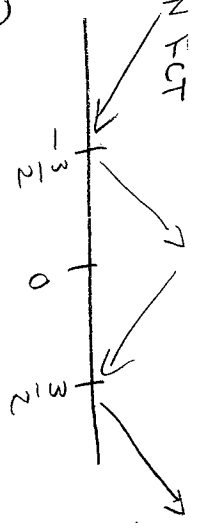
$\left(2\pi + \frac{\pi}{2}\right) r = \pi$
 $r = \frac{1}{2 + \frac{\pi}{2}} = \frac{2}{2 + \pi}$

$A'' = 2\pi + \frac{\pi}{2} > 0 \Rightarrow \cup \Rightarrow \text{min}$

Question: $r = 0 \Rightarrow x = 1$ $A = 1 \text{ sq in}$
 $2\pi r = 4 \Rightarrow r = \frac{4}{2\pi}$ $A = \pi \left(\frac{4}{2\pi}\right)^2 = \frac{4}{\pi} = \frac{16}{\pi} \text{ sq in} = \text{MAX}$

$x = 0, r = \frac{2}{\pi}$
 $\frac{16}{\pi} \text{ sq in} = \text{MAX}$

4. $y = 2x^4 - 9x^2$ EVEN FCT



$$y = 2x^4 - 9x^2 = 8x^3 - 18x$$

$$2x(4x^2 - 9) = 0$$

$$2x(2x-3)(2x+3) = 0$$

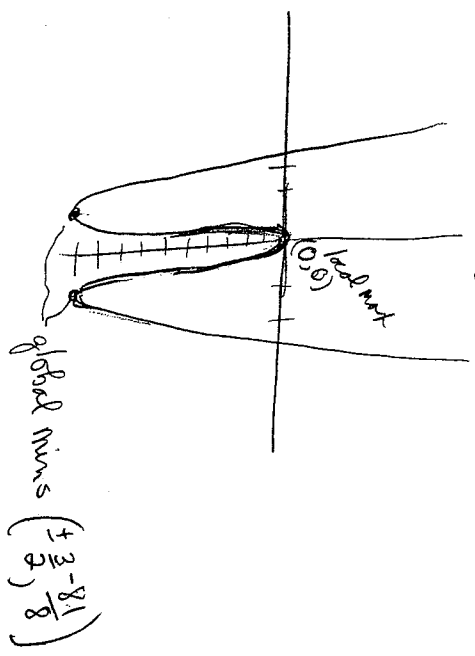
$$CR: x=0, \pm \frac{3}{2}$$

max at $(-\frac{3}{2}, 0) \cup (\frac{3}{2}, \infty)$
 dec at $(-\infty, -\frac{3}{2}) \cup (0, \frac{3}{2})$

a) $y'' = 24x^2 - 18 = 6(4x^2 - 3) = 6(2x - \sqrt{3})(2x + \sqrt{3})$



$y(0) = 0$
 $y(\frac{3}{2}) = y(-\frac{3}{2}) = 2(\frac{9}{16}) - 9(\frac{9}{4}) = \frac{81}{16}(2-4) = -\frac{81}{8}$



5. $V = \frac{\pi}{3} r^2 h$
 $\frac{r}{h} = \frac{5}{6}$

$$V = \frac{\pi}{3} r^2 \left(\frac{6r}{5}\right) = \frac{2\pi}{5} r^3$$

$$\frac{dV}{dt} = \frac{6\pi}{5} r^2 \frac{dr}{dt}$$

$$\frac{95 \cdot 50}{6\pi \cdot 100} = \frac{dr}{dt}$$

$$\frac{15}{4\pi} = \frac{45}{12\pi} \text{ m/min} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{5} \frac{dr}{dt}$$

$$\frac{dr}{dt} \Big|_{h=4} = \frac{3}{5} \frac{15}{4\pi} = \frac{9}{2\pi} \text{ m/min}$$

$$\boxed{\frac{9}{2\pi} \text{ m/min}}$$

$h = \frac{6}{5} r$
 $\frac{r}{h} = \frac{5}{6}$
 $r = \frac{10}{3}$