APPM 1350 **Make-up Exam #3** **FALL 2008**

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. On the front of your bluebook write (1) your name and the exam you are making up, *i.e.* "Exam #3", (2) your instructor's name, and (3) when your lecture meets. Also make a scoring table, with places for 5 problems, plus a total score. This exam has 5 problems. **Work all problems. Start each problem on a new page.** Box your answers. **SHOW ALL WORK.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

---

1. (20 points)

   (a) Consider a given function, say $f(x)$. Without explicitly calculating $f^{-1}(x)$, how can you tell whether $f(x)$ has an inverse?

   (b) Now suppose $f(x) = 2 - 3e^x$. Without explicitly calculating $f^{-1}(x)$, use your reasoning to show that $f(x)$ will have an inverse.

   (c) Now find $f^{-1}(x)$ and state the domain and range of $f^{-1}(x)$.

   (d) Find the *average value* of $f(x) = 2 - 3e^x$ over the interval $[0, 4]$.

2. (15 points) With approximately what velocity do you enter the water if you dive from a 10-m platform? (Use $g = 9.8$ m^2/sec. Be sure to include units in your answer.)

3. (25 points)

   (a) Write out the general *Trapezoidal Rule* for approximating the value of $\displaystyle\int_a^b f(x)dx$ with n intervals.

   (b) Given that $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx = \pi$, what estimate of π do you get if you numerically evaluate the definite integral using a Trapezoidal Rule with 2 intervals?

   (c) Suppose you want to estimate π to within 0.0006. How many intervals do you need so that the Trapezoidal Rule gives you an estimate within that tolerance? [*Note:* $|E_T| \le \dfrac{b-a}{12}h^2M$ and if $f(x) = \frac{4}{1+x^2}$, then $|f'(x)| \le \frac{8}{3}$ and $|f''(x)| \le 8$ on the interval [0,1].]

4. (20 points)

   (a) Evaluate $\displaystyle\int_0^{\sqrt{\ln(\pi)}} 2xe^{x^2}\cos(e^{x^2})\,dx$

   (b) Find the derivative of $y = \dfrac{\theta\sin(\theta)}{\sqrt{\sec(\theta)}}$ [*Hint:* Use *logarithmic differentiation.*]

   (c) Evaluate $\displaystyle\int \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

PLEASE TURN OVER. PROBLEM #5 ON THE OTHER SIDE.

5. (20 points)

(a) State **both** parts of the *Fundamental Theorem of Calculus.* Be sure to include the hypothesis of the theorem in your answer.

(b) Is $F(x) = \sqrt{x}\sin(x)$ an *anti-derivative* of $f(x) = \sqrt{x}\cos(x) + \dfrac{\sin(x)}{2\sqrt{x}}$? Why or why not?

(c) Find $\displaystyle\int_{\pi/2}^{\pi} \left[\sqrt{x}\cos(x) + \frac{\sin(x)}{2\sqrt{x}}\right] dx.$

Did you write "EXAM 3" on the front of your bluebook?

Now that you have finished the exam, please decide whether to turn it in or to walk away with it. If you turn in your finished exam, your score on this exam will replace your previous score on Exam 3. If you do not turn it in, your current score on Exam 3 will stand.