APPM 1350 FALL 2008 Make Up Exam #3 Soln's

1(a) $f(x)$ has an inverse if $f(x)$ is 1-1, i.e. if for each y in the range of f there is a unique x in the domain of f.

(b) $f'(x) = -3e^x < 0$, so $f(x)$ strictly decreasing implies $f(x)$ passe horizontal line test so $f(x)$ has an inverse.

(c) $f(x) = 2-3e^x$, so $y = 2-3e^x$ thus $e^x = \frac{y-2}{-3}$

so $x = \ln\left(\frac{y-2}{-3}\right)$, thus

$$f^{-1}(x) = \ln\left(\frac{x-2}{-3}\right) \quad D: (-\infty, 2) \quad R: (-\infty, \infty)$$

(d)
$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{4-0}\int_0^4 2-3e^x\,dx$$

$$= \frac{1}{4}\left(2x-3e^x\right)\Big|_0^4 = \frac{1}{4}\left[(8-3e^4)-(0-3e^0)\right]$$

$$= \frac{11-3e^4}{4}$$

2. $a(t) = v'(t) = 9.8 \Rightarrow v(t) = 9.8t + C_1$, at $(0,0)$ we have $C_1 = 0 \Rightarrow s'(t) = v(t) = 9.8t \Rightarrow s(t) = 4.9t^2 + C_2$, at $(0,0)$ we have $C_2 = 0 \Rightarrow s(t) = 4.9t^2$.

Now $s(t) = 10 \Rightarrow t^2 = 10/4.9 \Rightarrow t = \sqrt{10/4.9}$

Now finally $v\left(\sqrt{10/4.9}\right) = 9.8\sqrt{10/4.9}$

$$= 2(4.9)\frac{\sqrt{10}}{\sqrt{4.9}} = 2\sqrt{4.9}\sqrt{10} = 14 \text{ m/sec.}$$

3 (a) To approximate $\int_a^b f(x)\,dx$ with n subintervals use

$$T = h/2\left(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n\right)$$

where $h = b-a/n$, $y_k = f(x_k)$, $k = 0,1,\dots,n$.

(b) $\int_a^b f(x)\,dx = \int_0^1 \frac{4}{1+x^2}\,dx$, $n = 2$,

$$\begin{array}{ccc} 0 & 1/2 & 1 \\ \shortparallel & \shortparallel & \shortparallel \\ x_0 & x_1 & x_2 \end{array}$$

$h = b-a/2 = 1/2$, $h/2 = 1/4$ so,

$$T = \frac{1}{4}\left(f(0) + \overbrace{2f(1/2)}^{2\cdot f(1/2)} + f(1)\right) = \frac{1}{4}\left(4 + 2\left(\frac{16}{5}\right) + 2\right)$$

$$= \frac{1}{4}\left(\frac{20}{5} + \frac{32}{5} + \frac{10}{5}\right) = \frac{62}{20} = \frac{31}{10} = \underline{\underline{3.1}}$$

(c) $|E_T| \le \frac{b-a}{12} h^2 M = \frac{1}{12}\left(\frac{1}{n}\right)^2 8$

Need n s.t. $\frac{8}{12n^2} \le .0006 = \frac{6}{10000}$

so need n s.t. $n^2 \geq \frac{10000}{6} \cdot \frac{8}{12} = \frac{10000}{9}$

need $n \geq \sqrt{\frac{10000}{9}} = \frac{100}{9} \approx 34$

4 (a) $\int_0^{\sqrt{\ln(\pi)}} 2x e^{x^2} \cos(e^{x^2})\,dx \underset{\substack{u = e^{x^2} \\ du = 2xe^x dx}}{=} \int_{u(0)}^{u(\sqrt{\ln(\pi)})} \cos(u)\,du = \int_1^{\pi} \cos(u)\,du$

$= \sin(u)\Big|_1^{\pi} = \sin(\pi) - \sin(1) = -\sin(1).$

4 (b) $y = \frac{\theta \sin\theta}{\sqrt{\sec\theta}} \Rightarrow \ln(y) = \ln(\theta) + \ln(\sin\theta) - \frac{1}{2}\ln(\sec\theta)$

$$\frac{y'}{y} = \frac{1}{\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{2}\cdot\frac{\sec\theta\tan\theta}{\sec\theta}$$

So, $y' = \frac{\theta\sin\theta}{\sqrt{\sec\theta}}\left[\frac{1}{\theta} + \cot\theta - \frac{\tan\theta}{2}\right]$

(c) $\int \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$ Let $u = 1+\sqrt{y}$

$du = \frac{1}{2\sqrt{y}}\,dy$

So $\int \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \int \frac{du}{u^2} = \int u^{-2}\,du$

$= -u^{-1} + C$

$= \frac{-1}{1+\sqrt{y}} + C$

5. (a) Part 1

If f is continuous on $[a,b]$, then $F(x) = \int_a^x f(t)\,dt$ has a derivative at every pt. of $[a,b]$ and

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x), \quad a \le x \le b.$$

Part 2 If f is continuous at every pt. of $[a,b]$ and F is any antiderivative of f on $[a,b]$ then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

(b) Yes, since $[\sqrt{x}\sin(x)]' = \frac{\sin(x)}{2\sqrt{x}} + \sqrt{x}\cos(x) = f(x)$.

(c) $\int_{\pi/2}^{\pi} \sqrt{x}\cos(x) + \frac{\sin(x)}{2\sqrt{x}}\,dx = \sqrt{x}\sin(x)\Big|_{\pi/2}^{\pi} = \sqrt{\pi}\sin(\pi) - \sqrt{\frac{\pi}{2}}\sin(\pi/2) = -\sqrt{\frac{\pi}{2}}$