INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 7 problems, plus a total score. This exam has 7 problems, each worth 25 points. **Work any 6 problems. Clearly mark the problem you are skipping on the front of your bluebook, or we will grade problems 1-6.** Start each problem on a new page. Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

1. Evaluate the following limits and derivatives:

   (a) $\lim_{x\to\infty}(\ln(x^2))^{\frac{1}{5x}}$ (b) $\lim_{x\to -1}\frac{x+1}{|x+1|}$ (c) $\frac{d}{dx}\left[x^{\sin x}\right]$ (d) $\frac{d}{dx}\int_0^{\cos x}\frac{t}{\sqrt{2-t^2}}\,dt$

2. Evaluate the following integrals:

   (a) $\int_{-1/2}^{-1/2}\frac{t}{\sqrt{1-t^2}}\,dt$ (b) $\int_0^{1/4}\frac{dt}{\sqrt{1-4t^2}}$ (c) $\int_2^4\frac{dx}{x(\ln(3x))^5}$

3. For each of the following, determine if the statement is **Always True** or **Not Always True**. If the statement is not always true, explain why or give a counter-example.

   (a) There is some function $f(x)$ such that $0 < f'(x) \le 4$ for all x, $f(0) = -1$, and $f(2) = 9$.

   (b) If $g''(3) < 0$ and $g'(3) = 0$ then $g(x)$ has a local maximum at $x = 3$.

   (c) $\frac{d}{dx}\left[e^{-4x}\right]$ is negative for all $x > 0$.

   (d) If $h(x)$ is continuous and decreasing on the interval $[a, b]$ then the Trapezoidal Rule will overestimate $\int_a^b h(x)dx$.

   (e) If a function is differentiable at $x = a$ then it is continuous at $x = a$.

   (f) The function $y = \ln(x)$ is $o(\ln(x^2+1))$ as $x \to \infty$.

4. Optimization:

   (a) Find the maximum of $y = x^3 - 6x^2 + 9x - 2$ on the interval $[2, 4]$. Where does the maximum occur? Justify your answer.

   (b) You are designing a poster to contain 50 in^2 of printing with margins of 4 inches each at top and bottom and 2 inches at each side. What overall dimensions will minimize the amount of paper used?

5. Let $f(x) = \frac{1}{(x-2)^2}$

   (a) What is the average value of $f(x)$ on the interval $[-2, 1]$?

   (b) Since $f(x)$ is continuous on $[-2, 1]$, the Mean Value Theorem for Definite Integrals guarantees the existence of some c in this interval. Find c.

   (c) Using the definition of derivative, find $f'(x)$.

   (d) Find the linearization of $f(x)$ at $x = 0$, and use the linearization to estimate $f(0.1)$.

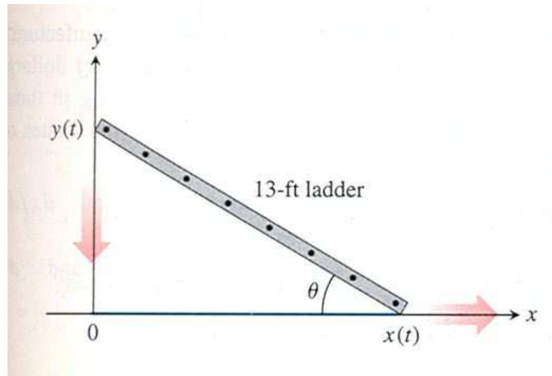
There are two more problems on the back!

6. A 13-ft. ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

   (a) How fast is the top of the ladder sliding down the wall then?

   (b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

   (c) At what rate is the angle θ between the ladder and the ground changing then?



7. Given the function $y = \dfrac{3x^2}{x^2 - 1}$

   (a) Sketch the graph of y. Be sure to identify (and label) any maxima, minima, inflection points, and asymptotes.

   (b) When is y increasing? When is it decreasing?

   (c) When is y concave up? When is it concave down?

   (d) Find the domain and range of y.

Formulae

The following equations may be useful:

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + C \text{ if } u^2 < a^2 \qquad \int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C \text{ if } u^2 > a^2$$