Final Exam 1350 Fall 08

1a) $\lim_{x\to\infty} (2\ln x)^{\frac{1}{5x}} = \infty^0$ indet

BUT

$\ln y = \lim_{x\to\infty} \frac{\ln(\ln x^2)}{5x} = \frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{2\ln x}\frac{2}{x}}{5} = \frac{0}{5} = 0$

$e^{\ln y} = e^0 \Rightarrow \boxed{y=1}$

b) $\lim_{x\to -1^+} \frac{x+1}{x+1} = 1$

$\lim_{x\to(-1)^-} \frac{x+1}{-(+x+1)} = -1$

RHL ≠ LHL

limit $\boxed{\text{DNE}}$

c) $\frac{d}{dx}\left(x^{\sin x}\right)$

$y = x^{\sin x}$

$\ln y = \sin x(\ln x)$

$\cancel{\frac{1}{y}}\frac{dy}{dx} = \boxed{\left[\frac{\sin x}{x} + (\ln x)(\cos x)\right] x^{\sin x}}$

d) $\frac{d}{dx}\int_0^{\cos x} \frac{t\,dt}{\sqrt{2-t^2}} = \frac{\cos t(-\sin t)}{\sqrt{2-\cos^2 t}}$

$= \boxed{\frac{-\cos t\sin t}{\sqrt{2-\cos^2 t}}}$

2a) $\displaystyle\int_{-1/2}^{-1/2} \frac{t}{\sqrt{1-t^2}}\,dt = \boxed{0}$ $\qquad \displaystyle\int_a^a f(x)\,dx = 0$

b) $\displaystyle\frac{1}{2}\int_0^{1/4} \frac{2\,dt}{\sqrt{1-4t^2}}$ $\qquad 4t^2 = u^2 \quad u(0) = 0$

$2t = u \quad u(\tfrac{1}{4}) = 1/2$

$2\,dt = du$

$$\frac{1}{2}\int_0^{1/2} \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \arcsin u \Big|_0^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\arcsin\tfrac{1}{2} - \cancelto{0}{\arcsin 0}\right] = \frac{1}{2}\frac{\pi}{6} = \boxed{\frac{\pi}{12}}$$

c) $\displaystyle\int_2^4 \frac{dx}{x(\ln 3x)^5}$ $\qquad u = \ln 3x$

$du = \dfrac{3}{3x}dx = \dfrac{dx}{x}$

$u(2) = \ln 6$

$u(4) = \ln 12$

$$\int_{\ln 6}^{\ln 12} \frac{du}{u^5} = \frac{-1}{4u^4}\Big|_{\ln 6}^{\ln 12}$$

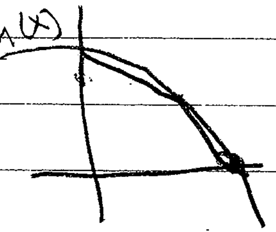
$$-\frac{1}{4}\left[\frac{1}{(\ln 12)^4} - \frac{1}{(\ln 6)^4}\right]$$

$$= \boxed{\frac{1}{4}\left(\frac{1}{(\ln 6)^4} - \frac{1}{(\ln 12)^4}\right)}$$

3a) Not Always True greatest possible increase is 10

$0 \le f'(x) \le 4 \Longrightarrow \text{inc'g} \le 8$

b) Always TRUE



c) ALways True

~~d)~~ Not Always True underestimate

~~e) Always True~~

~~f) Not always True~~ $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\ln(x^2+1)} \overset{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2x}{x^2+1}}$

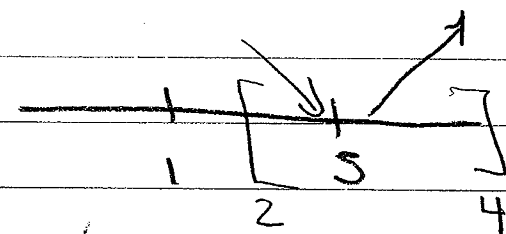
$\displaystyle = \lim_{x\to\infty} \frac{1}{x}\cdot\frac{x^2+1}{2x} = \lim_{x\to\infty} \frac{x^2+1}{2x^2} = \frac{1}{2}$

grow at the same rate

4. $y' = 3x^2 - 12x + 9 = 3(x^2-4x+3)$

$= 3(x-3)(x-1)$

CP: $x = 1, 3$



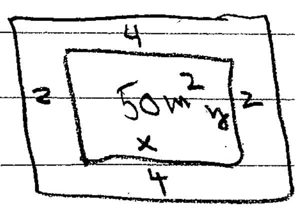
Check: $y(2) = 0$

$y(3) = -2$ (min)

$y(4) = 2$

ABS max on I is at $\boxed{(4,2) \quad \text{max} = 2}$

b)



$50 = xy$

$x = \frac{50}{y}$

MIN $A = (x+4)(y+8)$

$= \left(\frac{50}{y}+4\right)(y+8)$

$A = 50 + 400y^{-1} + 4y + 32$

$A' = -400y^{-2} + 4$

$\frac{400}{y^2} = 4 \Longrightarrow y^2 = 100$

$\boxed{y = \cancel{-10}, 10}$

$x = \frac{50}{10} = 5$

$\boxed{A'' = \frac{800}{y^3} > 0 \Rightarrow \cup \Rightarrow \text{MIN}}$

DIMENSIONS: $\boxed{9 \text{ in} \times 18 \text{ in}}$

$$\int u^{-2} = \frac{u^{-1}}{-1} = -\frac{1}{u}$$

5. $y = \frac{1}{(x-2)^2}$

a)

$$Avg = \frac{1}{3}\int_{-2}^{1} \frac{dx}{(x-2)^2} \longrightarrow \frac{1}{3}\int_{-4}^{-1} \frac{du}{u^2}$$

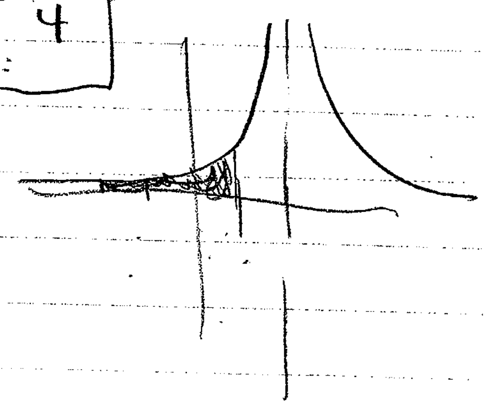
$u = (x-2)$

$du = dx$

$u(-2) = -4$

$u(1) = -1$

$$Avgf = \frac{1}{3}\left(-\frac{1}{x-2}\right)\Bigg|_{-2}^{1} = \frac{1}{3}\left[1 - \frac{1}{4}\right] = \boxed{\frac{1}{4}}$$



b)

$$\frac{1}{(c-2)^2} = \frac{1}{4}$$

$$(c-2)^2 = 4$$

$|c-2| = 2$ $\quad -c+2 = 2$ $\quad 0 \in [-2, 1]$

$c = 4$ $\quad -c = 0$

$$\boxed{c = 0}$$

$x+h-2$

$x+h-2$

c)

$$\lim_{h\to 0} \frac{\frac{1}{(x+h-2)^2} - \frac{1}{(x-2)^2}}{h}$$

$$= \lim_{h\to 0} \frac{(x-2)^2 - (x+h-2)^2}{h(x-2)^2(x+h-2)^2}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} - \cancel{4x} + \cancel{4} - (\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{4})}{h(x-2)^2(x+h-2)^2}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}(2x+h-4)}{\cancel{h}(x-2)^2(x+h-2)^2} = \frac{-(2x-4)}{(x-2)^4}$$

$$= \frac{-2(x-2)}{(x-2)^4} = \boxed{\frac{-2}{(x-3)^3}}$$

5d) $y' = \frac{-2}{(x-2)^3}$ $\qquad \frac{-2}{-8}$

$y'(0) = \frac{1}{4}$

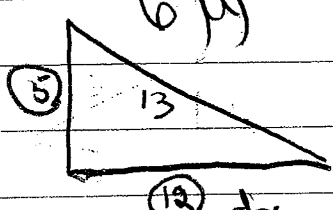
$y(0) = \frac{1}{4}$

$y - \frac{1}{4} = \frac{1}{4}(x - 0)$

$\boxed{y = \frac{1}{4}x + \frac{1}{4}}$ $\qquad \boxed{1.275}$ =

$y(0.1) \approx \frac{1}{4}\left(\frac{1}{10}\right) + \frac{1}{4}\frac{10}{10} = \boxed{\frac{11}{40}}$

6) a) find $\frac{dy}{dt}\Big|_{x=12}$ $\qquad x^2 + y^2 = z^2$



$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\cancel{\frac{dz}{dt}}$ $\quad 0$

$\frac{dx}{dt} = 5$ ft|sec

$5\frac{dy}{dt} = -12(+5)$

$\frac{dy}{dt} = \boxed{-12 \text{ ft}|\text{sec}}$

b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$

$= \frac{1}{2}(12)(-12) + \frac{1}{2}5(+5)$

$= \frac{-144}{2} + \frac{25}{2} = \boxed{\frac{-119}{2} \text{ ft}^2|\text{sec}}$

6c) $\sin\theta = \frac{y}{13}$

$\cos\theta \frac{d\theta}{dt} = \frac{1}{13}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{1}{13}(-12)\left(\frac{1}{\frac{12}{13}}\right)$

$= \frac{1}{\cancel{13}}\left(\frac{-\cancel{12}}{1}\right)\left(\frac{\cancel{13}}{\cancel{12}}\right) = \boxed{-1 \text{ rad/sec.}}$

7) $y = \frac{3x^2}{x^2-1}$

$$\begin{array}{r} 3 + \frac{3}{x^2-1} \\ x^2-1 \overline{)\,3x^2 } \\ 3x^2-3 \\ \hline 3 \end{array}$$

a) HA: $\boxed{y=3}$ $\lim_{x\to\infty}\frac{3x^2}{x^2-1} \overset{DOP}{=} 3$

VA: $\boxed{x=\pm1}$ ⓐ $\lim_{x\to1^+}\frac{3x^2}{x^2-1} = \frac{3}{+0} = +\infty$

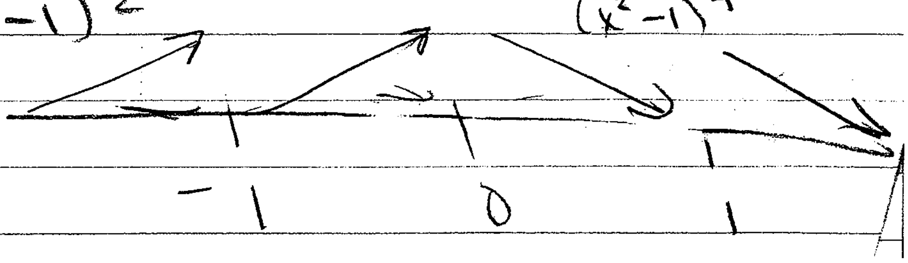
ⓑ $\lim_{x\to -1^-}\frac{3x^2}{x^2-1} = \frac{3}{+0} = +\infty$

OA: $\boxed{\text{NONE}}$ deg num = deg den.

Note: fct is EVEN $y(-x) = y(x)$

$y' = \frac{(x^2-1)(6x) - 3x^2(2x)}{(x^2-1)^2} = \frac{\cancel{6x^3} - 6x - \cancel{6x^3}}{(x^2-1)^4}$

$= \frac{-6x}{(x^2-1)^4}$



7 a) BELOW

b) $\boxed{\begin{array}{l} \text{inc'g } (-\infty,-1) \cup (-1,0) \\ \text{dec'g } (0,1) \cup (1,\infty) \end{array}}$

c) $\boxed{\begin{array}{l} \text{CU } (-\infty,-1) \cup (1,\infty) \\ \text{CD } (-1,1) \end{array}}$

d) $\boxed{\begin{array}{l} \text{Dom} f = \{x \mid x \neq \pm 1\} \\ \text{Rg} f = (-\infty,0) \cup (3,\infty) \end{array}}$

a)