INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) student ID, and (3) instructor's name on the front of your bluebook. Also make a scoring table, with places for 7 problems, plus a total score. This exam has 7 problems each worth 25 points. **Work any 6 problems. Mark clearly on the front of your bluebook which exam problem you are skipping.** (Otherwise we will grade the first 6 problems.) Start each problem on a new page. $\boxed{\text{Box}}$ your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

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1. Perform the following operations.

   (a) $\displaystyle\int \frac{dy}{y^2+6y+10}$ (b) $\displaystyle\int_0^{\frac{3\sqrt{2}}{4}} \frac{ds}{\sqrt{9-4s^2}}$

   (c) Given that $\alpha = \sec^{-1}(-\sqrt{13}/2)$, find $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$.

2. Evaluate the following limits.

   (a) $\displaystyle\lim_{x\to\infty} \frac{1}{x\ln(x)} \int_1^x t^{-1}dt$ (b) $\displaystyle\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$ (c) $\displaystyle\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x$

   (d) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$ (e) $\displaystyle\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2(x)}$

3. Optimization

   A right circular cylinder (*i.e.*, a barrel) is to be enclosed in a sphere of radius $\sqrt{3}$.

   (a) Find the height and radius of the cylinder with the largest possible volume.
   (b) Show that your result is a maximum.
   (c) What is the maximal volume of the barrel?

4. The following question relates to the following three functions:

   i. e^x ii. x^x iii. $e^{x/2}$

   (a) Order the three functions from slowest growing to fastest growing as $x \to \infty$. Justify your answers.
   (b) For each of the three functions, if it has a finite limit as $x \to 0^+$, find that limit. If it has an infinite limit or if it has no limit as $x \to 0^+$, say so and explain why.
   (c) Find the derivative of each function and evaluate each derivative at $x = 1$.
   (d) Sketch the graph of $y = x^x$ on the interval $(0, \infty)$. Be sure to identify any maxima, minima, and / or asymptotes on your graph. Also identify where the graph is concave up and where it is concave down.

There are three more problems on the back!

5. Integrals

   (a) Find the average value of $y = \dfrac{6}{9+x^2}$ on the interval $[0, \sqrt{3}]$.

   (b) Find the average value of $y = \dfrac{6x}{9+x^2}$ on the interval $[0, \sqrt{3}]$.

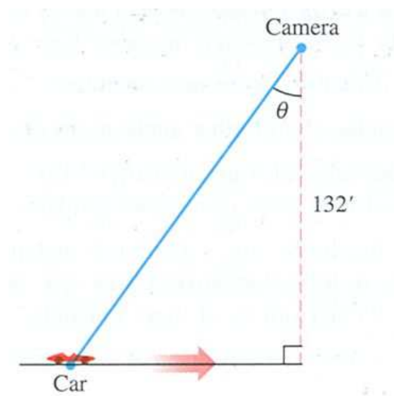
   (c) Find the total area between the curve $y = \dfrac{\ln x}{x}$ and the x-axis on the interval $[1/e, e]$.

6. Related Rates

   (a) You are videotaping a race from a stand 132 ft from the track, following a car that is moving at 180 mph (264 ft/sec). How fast will your camera angle θ be changing when the car is right in front of you? A half second later? (Be sure to include units.)

   (b) The camera angle θ can be written in terms of an inverse trigonometric function involving the position of the car and the distance from the stand to the track. What is the relation?

   (c) Find the rate of change of camera angle as a function of the car's position and its speed.



7. $\ln 2$: Recall that $\ln x = \displaystyle\int_1^x \frac{dt}{t}$ for $x > 0$.

   (a) Use the trapezoidal rule with $N = 3$ equal subintervals to find an approximate value of $\ln 2$. Your answer should be a number, not a complicated expression.

   (b) The error estimate for the trapezoidal rule with uniform spacing is $|E_T| \leq \dfrac{b-a}{12}h^2 M$. Based on this estimate, how far from the correct value of $\ln 2$ could your result from part (a) be? Give the maximum possible deviation.

   (c) How many subintervals would be required to approximate $\ln 2$ with error $\leq \dfrac{1}{6} \cdot 10^{-6}$?

   (d) Note that $y = \ln 2$ solves the equation $e^y = 2$. Starting at $y_0 = 0$, take 2 steps of Newton's method to find a *second* approximation of $\ln 2$. Find y_1 and y_2 explicitly.

The following equations may be useful:

Volumes

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{cylinder} = \pi r^2 h$$

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

Integrals

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \text{ if } u^2 < a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C \text{ if } u^2 > a^2$$