Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. You need to solve all 8 problems in order to receive full credit. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points)

   (a) State the definition of the derivative of a function f at a point $x = a$.

   (b) Using the definition at point (a), calculate the derivative of $f(x) = \dfrac{1}{\sqrt{x+3}}$ at the point $x = 1$.

   (c) Using the linearization of f at $x = 1$, estimate the value of $\dfrac{1}{\sqrt{4.1}}$.

2. (25 points) Calculate the following limits (if they do not exist, explain why).

   (a) $\displaystyle\lim_{x\to 3} \frac{x^2-9}{\sqrt{3}-\sqrt{x}}$

   (b) $\displaystyle\lim_{x\to \infty} \frac{2x^3-2x+4}{5x^3+x}$

   (c) $\displaystyle\lim_{x\to 1^+} \frac{x^2+1}{x-1}$

   (d) $\displaystyle\lim_{x\to 0^+} x\ln x$

3. (20 points) Calculate $\dfrac{dy}{dx}$ for the following functions, defined on $x > 0$. Simplify your answer when possible.

   (a) $y = \dfrac{\ln x}{\ln x + \ln 2}$

   (b) $y = e^{x^2+\sqrt{x}}$

   (c) $y = x^{\sqrt{x+1}}$

4. (25 points) Consider the function $f(x) = \dfrac{x^2-3}{x-2}$, for $x \neq 2$.

   (a) Calculate $f'(x)$ and $f''(x)$.

   (b) Find the critical points of f, and the intervals on which f is increasing and decreasing, respectively.

   (c) Find the inflection points of f, and the intervals on which f is concave up and concave down, respectively.

   (d) Find the horizontal, oblique and vertical asymptotes of f (if applicable). Explain your answers carefully (by calculating the appropriate limit).

   (e) Find the x and y-intercepts of the graph of f.

   (f) Using the information from all previous points, sketch the graph of f. Label the asymptotes, the critical points and the intersepts with the axes.

5. (10 points) The following two parts are not related to each other. Chose to solve one of the two.

   (a) Verify that the point $P = (\pi/4, \pi/2)$ is on the curve $x\sin(2y) = y\cos(2x)$. Find the tangent line to the curve at P.

(b) Use Newton's method to estimate the only real solution of $x^3+3x+1=0$. Start with $x_0 = 0$ and find x_2. Using only the derivative $f'(x)$, roughly sketch the graph of the function f, and show on this graph how you got the point x_1 from x_0.

6. (10 points) The following two parts are not related to each other. Chose to solve one of the two.

   (a) Consider the function $f(x) = \dfrac{x^2+x-1}{x+2}$. Using the Intermediate Valus Theorem, show that there is a value $x = c$ for which $f(c) = 1$.

   (b) Find the function f and the constant $a > 0$ that satisfy the following equation for all values of $x > 0$:

$$\int_a^x \frac{f(t)}{t}dt = x^2 - a$$

7. (30 points) Calculate the following integrals:

   (a) $\displaystyle\int_0^{\pi/2} \cos^3 x \sin t dt$

   (b) $\displaystyle\int_0^1 \sqrt{1-x^2}dx$

   (c) $\displaystyle\int \frac{\sqrt{x}}{x+1}dx$ (Hint: Start with a substitution $u = \sqrt{x}$.)

8. (25 points) The evolution in time (measures in minutes) of the temperature $T = T(t)$ (in ^{o}C) of an warm pie placed in a refrigerator with temperature $T_s = 1^o$C is given by the differential equation:

$$\frac{dT}{dt} = -k(T - T_s)$$

   where k is a cooling constant.

   (a) **Extra credit (10 points)** Solve the separable differential equation to show that, if the pie has $T(0) = T_0$ when placed in the fridge, the temperature of the pie as a function of time is given by:

$$T(t) = T_s + e^{-kt}(T_0 - T_s)$$

   (b) Using the solution from (a), find the cooling constant k for the pie to cool of from $T_0 = 80^o$C to 50^oC in 20 minutes. Do not simplify your numerical answer.

   (c) With the same cooling constant, how long will it take the pie to get to 20^oC? Do not simplify your numerical answer.

   (d) What is $\lim_{t\to\infty} T(t)$.

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Useful trig formulas you can use:

$$\int \frac{1}{\sqrt{1-u^2}}du = \sin^{-1}(u) + C \qquad \int \frac{-1}{\sqrt{1-u^2}}du = \cos^{-1}(u) + C$$

$$\int \frac{1}{1+u^2}du = tan^{-1}(u) + C$$