APPM 1350 **Midterm III** **Spring 2009**

Books, notes and electronic devices are not permitted. Write your name and recitation number on the front of your bluebook. There are **5 problems**, *plus an extra credit problem. Show your work clearly and box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

1. **(20 points)** Consider the integral $\int_1^3 \frac{1}{x} dx$.
   (a) Estimate the value of the integral using $n = 4$ rectangles and left-hand end-points (you do not need to simplify your answer).
   (b) Estimate the same integral using the trapezoidal sum for $n = 4$ (you do not need to simplify your answer).
   (c) Which of the above is an underestimate/overestimate? Justify your answer.(Hint: you may want to sketch the graph of the function, together with the respective rectangles and trapezoids, and explain.)
   (d) How large do you have to make n to be sure that the corresponding trapezoidal estimate is within $3 * 10^{-4}$ of the real value of the integral?

   Hint: The trapezoidal sum and error bound for the trapezoidal rule are
   $T_n = \frac{h}{2}(y_0 + 2y_1 + ... + 2y_{n-1} + y_n)$
   $|E_T| \leq \frac{b-a}{12} h^2 M$

2. **(20 points)** Calculate the integrals:
   (a) $\int \frac{\sqrt[3]{v} - 1}{\sqrt{v}} dv$
   (b) $\int_{\pi/3}^{\pi/2} \sin(\theta) \cos^3(\theta) d\theta$
   (c) $\int_0^1 r^2 \sqrt{1 - r^3} dr$

3. **(20 points)** Consider the function: $h(x) = \cos x + \int_0^{\sqrt{x}} t^3 \sin t^2 dt$ on the interval $[0, \pi]$.
   (a) State the FTC, parts I and II.
   (b) Calculate the derivative $\frac{dh}{dx}$.
   (c) Find the critical points of h in the interval $[0, \pi]$.

4. **(20 points)** Consider the function $f(x) = \frac{1}{\pi}\sqrt{4 - x^2}$.

(a) Find the average value of the function f on the interval $[0, 2]$. (Hint: Interpret the integral as an area.)

(b) The Mean Value Theorem states that there exists at least one value $x = c$ between 0 and 2 such that $f(c)$ equals to the average value of f. Find the value(s) of c.

5. **(20 points)** Consider the function $f(x) = \dfrac{x}{x-1}$.

(a) What is the domain of f?

(b) Is f one-to-one? If yes, calculate the inverse function $f^{-1}(x)$.

(c) What are the domain and range of f^{-1}?

Extra-credit: (20 points)

A fence of height H is D feet away from a vertical wall. At what angle θ should a ladder be leaned against the fence in order that the minimum length ladder be required to stretch from the ground to the wall? Use the following guidelines:

(a) Where (how far from the fence) should the leg of the ladder A be placed on the ground so that the ladder makes a given angle θ with the fence? (Hint: your answer will depend on D and θ.)

(b) How far is A from the vertical wall, for the given angle θ?

(c) How long is the ladder for the given θ?

(d) The length L of the ladder depends on θ (via one of its trig functions $\sin\theta$, $\cos\theta$ etc). Differentiate L with respect to θ. (Hint: when differentiating, remember that H and D are constants that do not depend on θ.)

(e) Set the derivative of L equal to zero and solve for θ to find for which angle the minimum value of L is obtained. (Hint: your answer may still depend on H and D. You may express θ in the form of one of its trig functions, e.g. $\cos\theta = 100G + \sqrt[100]{H}$.)

Good luck!