APPM 1350 **FINAL EXAM** **SUMMER 2009**

On the front of your bluebook write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. SHOW ALL YOUR WORK in your bluebook and $\boxed{\text{BOX}}$ in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. **Please start each new problem on a new page of the bluebook.**

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1. (50 points, 5 points each) For each of the follwing unrelated statements, answer TRUE or NOT NECESSARILY TRUE.

   (a) If $f'(x) \geq 0$ for all x and $f(0) = 0$, the $f(x) \geq 0$ for all x.

   (b) The function $f(t) = \dfrac{\sin t}{t^2}$ has a continuous extension at $t = 0$.

   (c) The derivative of the function $\csc^2 x$ is same as the derivative of $\cot^2 x$.

   (d) $\displaystyle\int_{-\pi}^{\pi} \frac{x \sin x}{x^2 + 1}\, dx = 0$

   (e) If f and g are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\displaystyle\int_a^b f(x)\, dx \geq \int_a^b g(x)\, dx$

   (f) There exists a function f such that $f(1) = -2$, $f(3) = 0$, and $f'(x) > 1$ for all x.

   (g) If g is continuous and $g(5) = 2$ and $g(4) = 3$ then $\displaystyle\lim_{x \to 2} g(4x^2 - 11) = 2$.

   (h) All continuous functions have derivatives.

   (i) All continuous functions have antiderivatives.

   (j) The inverse function of $y = e^{3x}$ is $y = \dfrac{1}{3}\ln(x)$

2. (25 points, 5 points each) Answer each of the following unrelated questions. No partial credit will be awarded, and no justification is necessary.

   (a) Find the average rate of change of $a(r) = log_{10}(r^2 + 1)$ on the interval $[0, 3]$.

   (b) Find the instantaneous rate of change of $a(r) = log_{10}(r^2 + 1)$ at $t = 3$.

   (c) Find all critical points of the function $h(t) = t + \dfrac{9}{t}$.

   (d) Evalutate: $\displaystyle\sum_{k=0}^{5} \sin \frac{(2k + 1)\pi}{2}$

   (e) State the Fundamental Theorem of Calculus, Parts I and II.

THE EXAM IS CONTINUED ON THE BACK

3. (30 points) Evaluate each of the following limits, if it exists. If the limit does not exist, state this and state your justification. Show all your work.

   (a) $\displaystyle\lim_{x\to 0} \frac{x \sin x}{1-\cos x}$

   (b) $\displaystyle\lim_{z\to 0^+} z^{1-z}$

   (c) $\displaystyle\lim_{h\to 0} \frac{1}{h}\int_0^h \frac{\arccos t}{1+t^2}\,dt$

4. (30 points) Find $\frac{dy}{dx}$ in each case. No simplification is necessary.

   (a) $y = (\arcsin x)\left[\ln(\arctan x)\right]$

   (b) $y = \dfrac{e^{x^2}}{x}$

   (c) $y = \dfrac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

5. (30 points) Evaluate each of the following integrals.

   (a) $\displaystyle\int (\tan x)^{-3/2} \sec^2 x\, dx$

   (b) $\displaystyle\int_0^{\ln 16} e^{t/4}\, dt$

   (c) $\displaystyle\int \frac{1}{x(1+(\ln x)^2)}\, dx$

6. (20 points)

7. (15 points)

Complete 2 of the following 3 problems (A, B, and C). Select either problem A, B, or C to be Q6 (worth 20 points), and one of the remaining two problems to be Q7 (worth 15 points). You MUST identify in your bluebook either A, B, or C for each of Q6 and Q7. Failure to do so will result in the **WORST-CASE** scenario point distribution!

A. If f is a continuous function such that

$$\int_0^x f(t)\,dt = xe^{2x} + \int_0^x e^{-t} f(t)\,dt$$

for all x, find an explicit formula for $f(x)$.

B. For what values of k does the function $y = e^{kx}$ satisfy the equation $y'' + 6y' + 8 = 0$?

C. Find the linearization of $f(x) = \ln(1+3x)$ around $c = 0$. Use it to give an approximate value of $\ln(1.03)$.

EXTRA CREDIT: (2 points each)

1. Name one of the places Erin traveled to over the summer. (3 options!)
2. Where did Brutz go when he missed recitation?
3. What is one of Erin's favorite cartoons? (2 options!)
4. How many people came to recitation on July 16?
5. Which end of the chalkboard do the erasers always end up on by the end of lecture?

Some Useful Information

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$
$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\sin A\sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$$
$$\cos A\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$
$$\sin A\cos B = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$
$$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\operatorname{arccsc} x = -\frac{1}{|x|\sqrt{x^2-1}}$$